Analytical Signal Integrity Verification Models for Inductance-Dominant Multi-Coupled VLSI Interconnect

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Outline

- Technical Trend and Problems
- TWA-Technique
- Multi-Coupled Lines
- Analytical Models and Verification
- Summary and Conclusion

Technical Challenge

Technical Trend

High-Speed, High-Density

- Inductance-Dominant System
- Longer and Tighter Spacing

Problems

More Complicated Signal Integrity Problems

Delay, X-talk, Ringing, Glitch

Accuracy?

Computation Time?

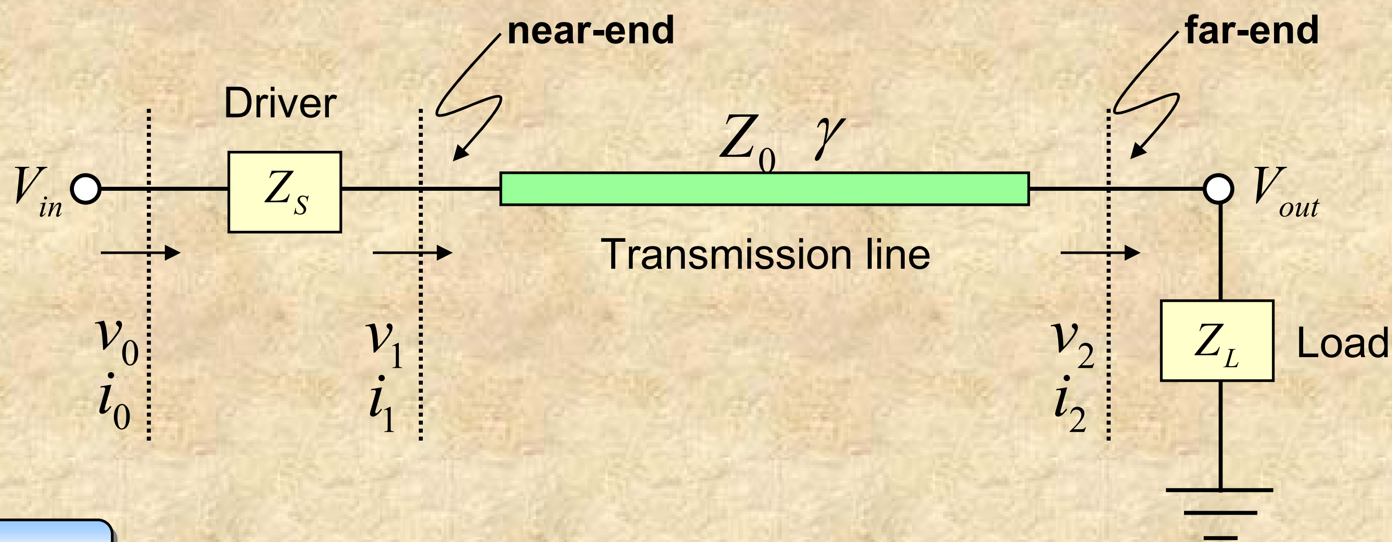
Stability?

Research Goal

- New Paradigm(TWA-Based)
- Fast, Accurate, Analytical Signal Integrity Models

Problems

System Function of a Single Transmission Line



Far-end

$$H_{far}(s) = \frac{v_2}{v_0} = \frac{1}{\cosh(\gamma\ell) + \frac{Z_0}{Z_L}\sinh(\gamma\ell) + \frac{Z_S}{Z_0}\sinh(\gamma\ell) + \frac{Z_S}{Z_L}\cosh(\gamma\ell)}$$

Near-end

$$H_{near}(s) = \frac{v_1}{v_0} = \frac{Z_0 Z_L \cosh(\gamma\ell) + Z_0^2 \sinh(\gamma\ell)}{Z_0(Z_S + Z_L)\cosh(\gamma\ell) + \left(Z_0^2 + Z_S Z_L\right)\sinh(\gamma\ell)}$$

Unit Step Response in a Single TL

$$V_{0-far}(s) = \frac{1}{s}\cdot H_{far}(s)$$
$$= \frac{1}{s\cdot\left\{\cosh(\gamma\ell) + \frac{Z_0}{Z_L}\sinh(\gamma\ell) + \frac{Z_S}{Z_0}\sinh(\gamma\ell) + \frac{Z_S}{Z_L}\cosh(\gamma\ell)\right\}}$$

$$v_{0-far}(t) = \mathcal{L}^{-1}\left\{\frac{1}{s}\cdot H_{far}(s)\right\}$$

$$V_{0-near}(s) = \frac{1}{s}\cdot H_{near}(s)$$
$$= \frac{Z_0 Z_L\cosh(\gamma\ell) + Z_0^2\sinh(\gamma\ell)}{s\cdot\left\{Z_0(Z_S+Z_L)\cosh(\gamma\ell) + (Z_0^2 + Z_S Z_L)\sinh(\gamma\ell)\right\}}$$

$$v_{0-near}(t) = \mathcal{L}^{-1}\left\{\frac{1}{s}\cdot H_{near}(s)\right\}$$

Require Numerical Integration!!

Dominant Pole Approximation(using 3-Dominant Poles)

Far-end

$$H_{far}(s)=\frac{v_2}{v_0}=\frac{1}{\cosh(\gamma\ell)+\frac{Z_0}{Z_L}\sinh(\gamma\ell)+\frac{Z_S}{Z_0}\sinh(\gamma\ell)+\frac{Z_S}{Z_L}\cosh(\gamma\ell)}$$

$$v_{0-far}(t)=\mathcal{L}^{-1}\left\{\frac{1}{s}\cdot H_{far}(s)\right\}\approx\mathcal{L}^{-1}\left\{\frac{1}{s(s-s_1)(s-s_2)(s-s_3)}\right\}$$

$$=\mathcal{L}^{-1}\left\{\frac{a_0}{s}+\frac{a_1}{s-s_1}+\frac{a_2}{s-s_2}+\frac{a_3}{s-s_3}\right\}\ \square\ v_{03-far}(t)$$

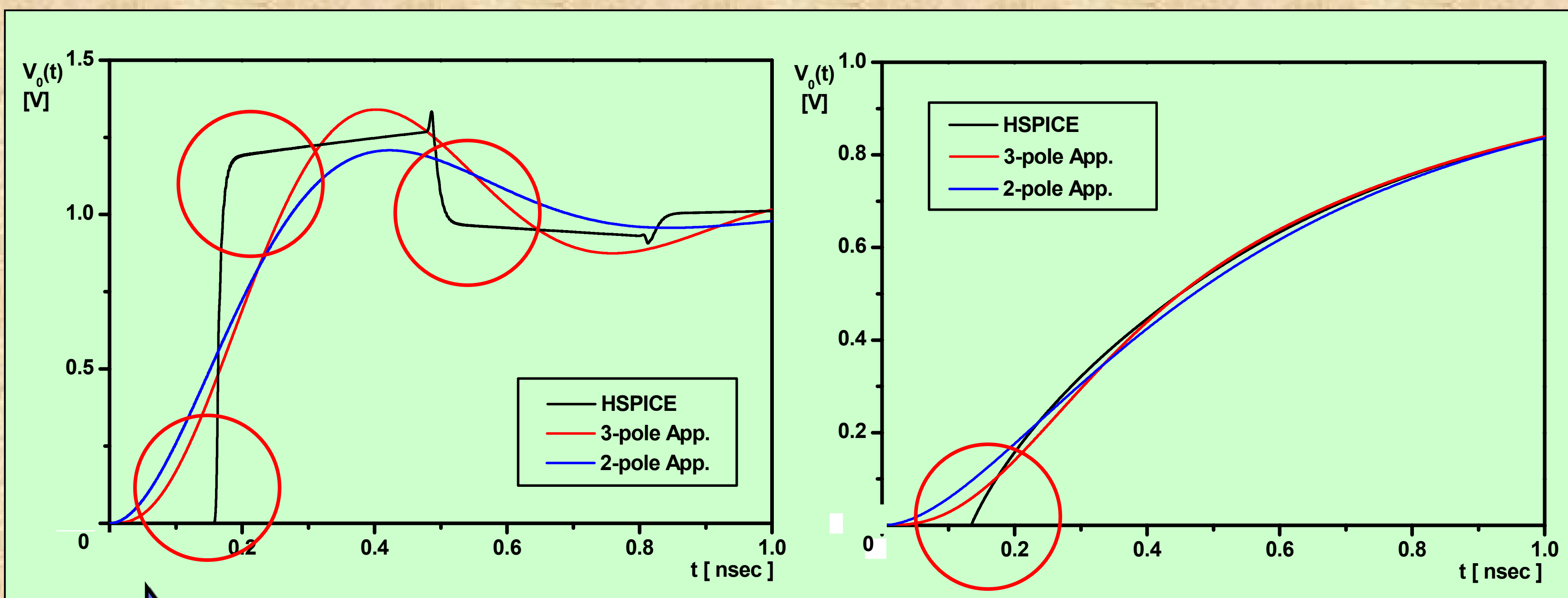
Do not Require Numerical Integration!!

Near-end

$$H_{near}(s)=\frac{v_1}{v_0}=\frac{Z_0Z_L\cosh(\gamma\ell)+Z_0^2\sinh(\gamma\ell)}{Z_0(Z_S+Z_L)\cosh(\gamma\ell)+(Z_0^2+Z_SZ_L)\sinh(\gamma\ell)}$$

$$v_{0-near}(t)=\mathcal{L}^{-1}\left\{\frac{1}{s}\cdot H_{near}(s)\right\}\approx\mathcal{L}^{-1}\left\{\frac{1}{s}\cdot\frac{q_1+q_2s+q_3s^2+q_4s^3}{p_1+p_2s+p_3s^2+p_4s^3}\right\}\ \square\ v_{03-near}(t)$$

Problems of 3-Pole Approximation

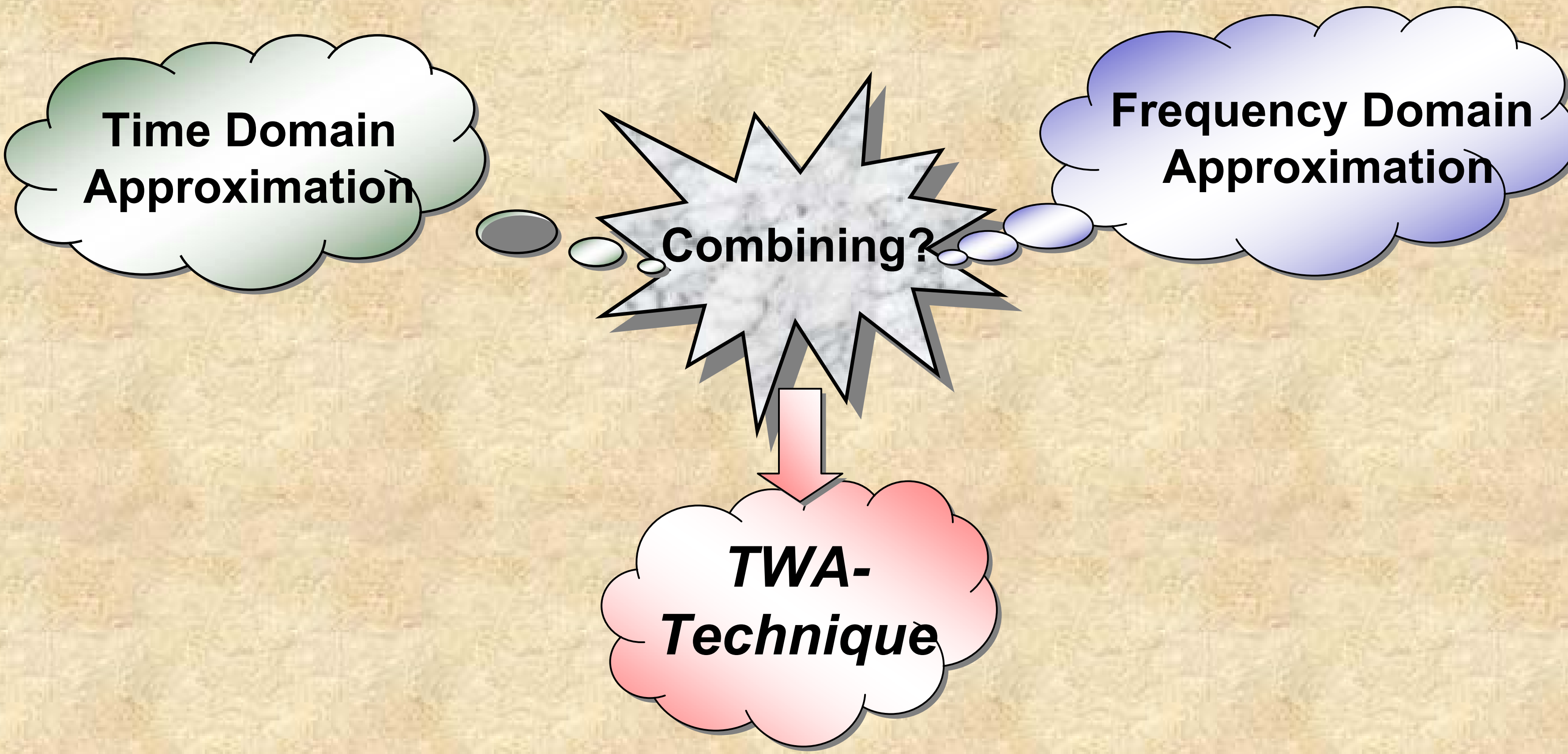


Dominant pole approximation does not reflect "Inductance-Dominant" effects.

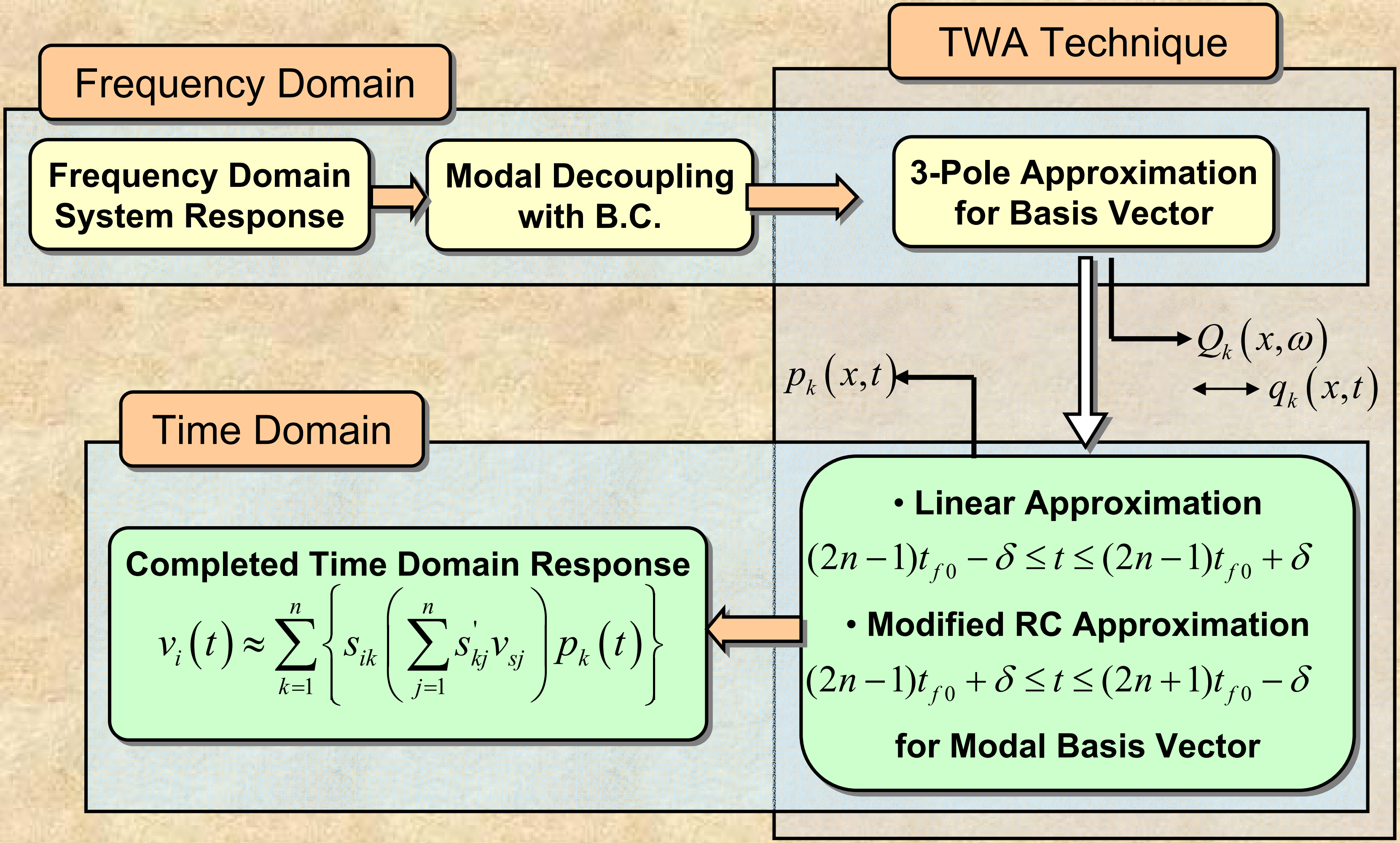
How to Incorporate the High-Frequency Effect?

Traveling-Wave-Based Waveform Approximation(TWA)

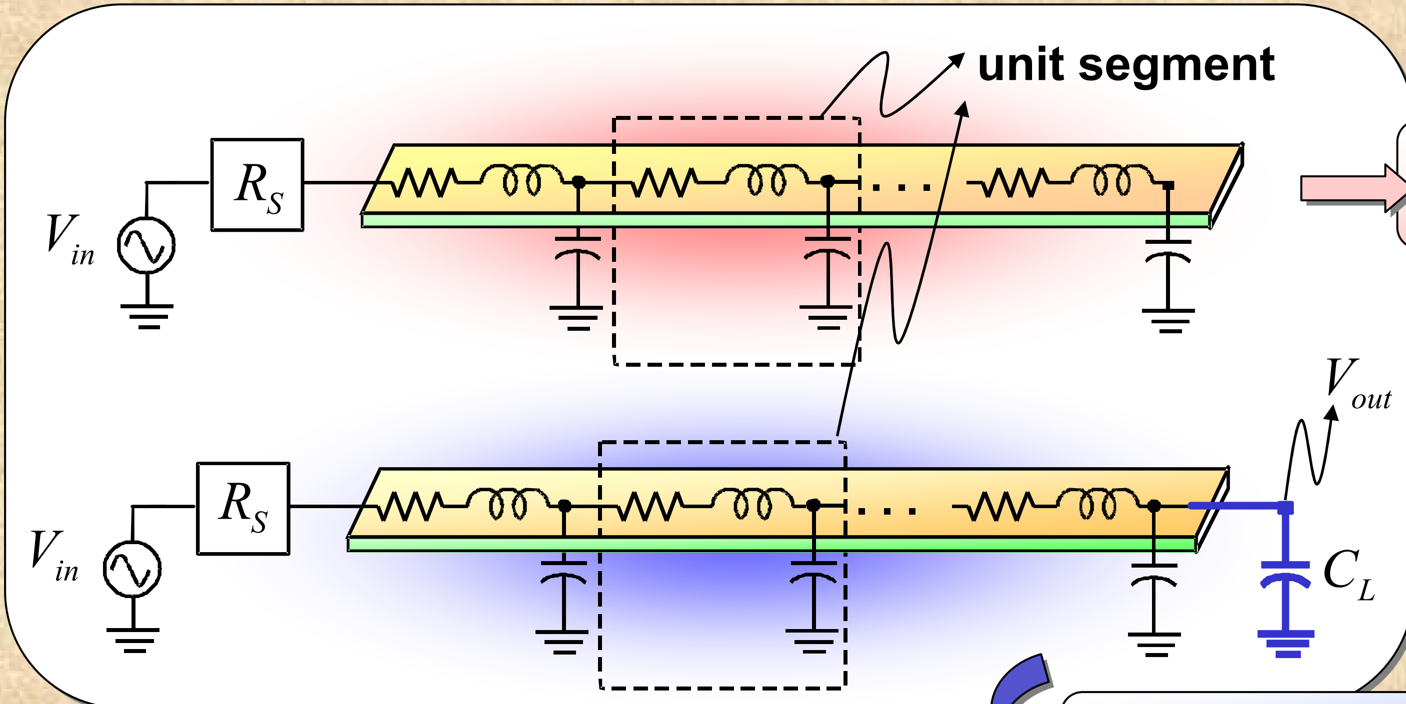
[Y. Eo, et al., “Traveling-wave-based ~,” will be published in IEEE T-CAD]



TWA-Based Time-Domain Transient Signal Characterizations



Effective Time of Flight : Loading Effect



$t_f^- = \sqrt{L_{line} C_{line}}$

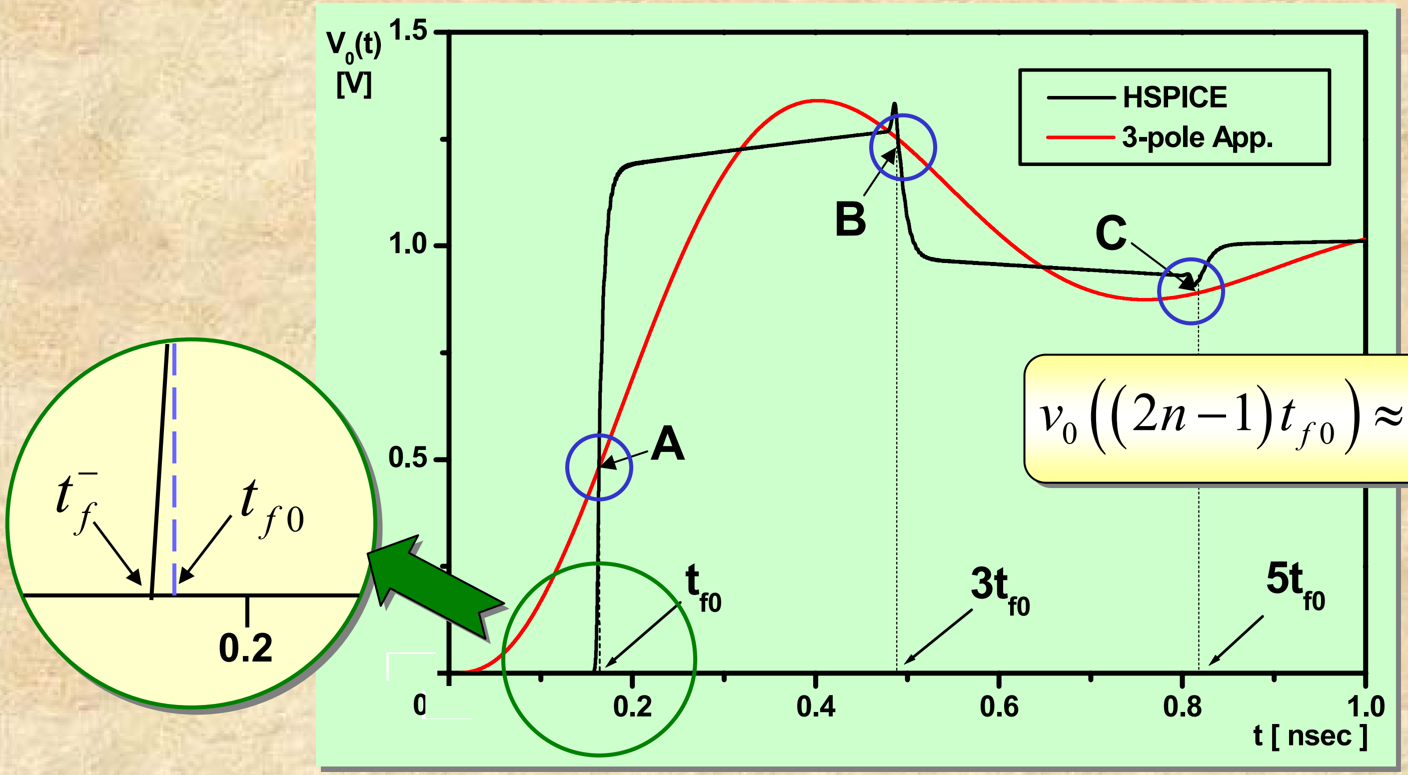
$t_{f0} \cong \sqrt{L_{line}\left(C_{line} + C_L\right)}$

Capacitance-Loading Effect

Increase the effective time of flight $\left(t_{f0}\right)$

Frequency-Domain Characteristics
(Low-Frequency Characteristics)

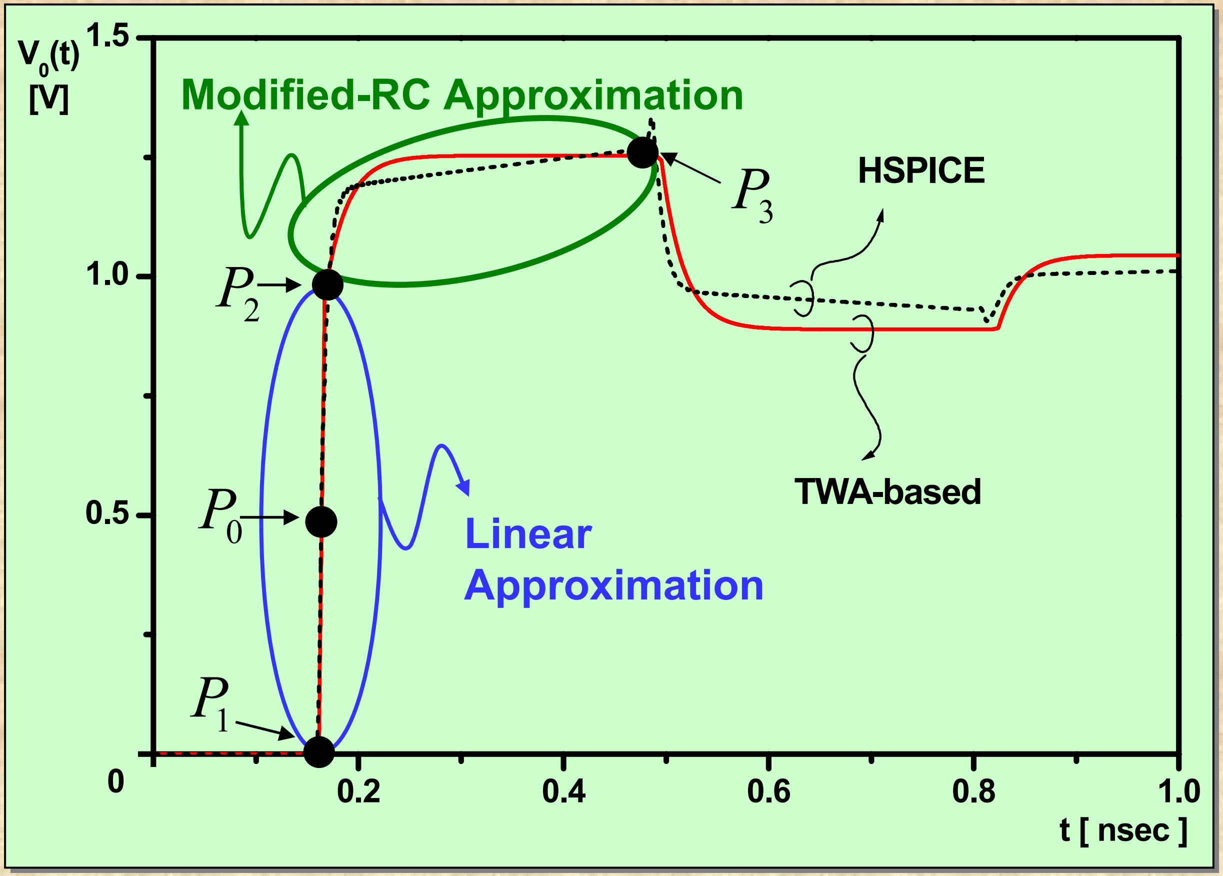
- **with 3-Pole-Based Frequency Domain Response**



$$v_0\left(\left(2n-1\right)t_{f0}\right) \approx v_{03}\left(\left(2n-1\right)t_{f0}\right)$$

- ***"Reflection" means "fast transient".***

Time-Domain Characteristics
(High-Frequency Characteristics)



We can determine the analytical form of expressions

Since we know two points.

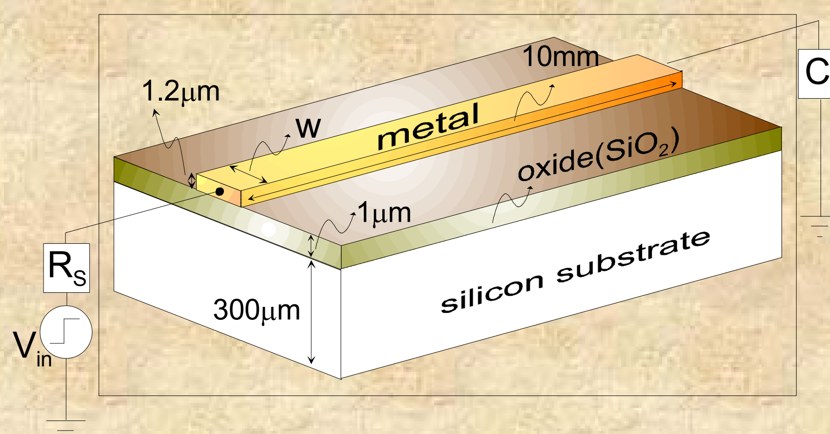
Verification of TWA in a Single Line

TL Parameters

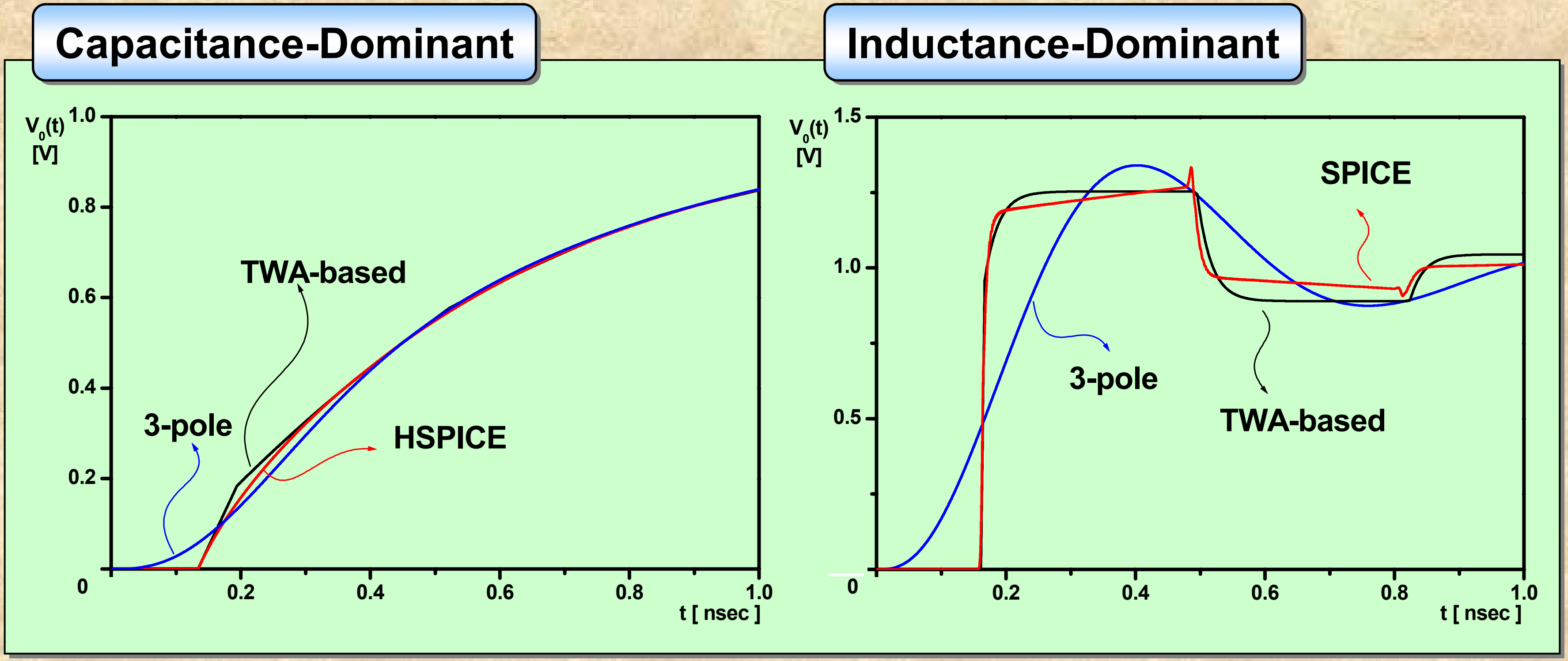
Capacitive

Inductive

Width [μm]	R [Ω/cm]	L [nH/cm]	C [pF/cm]	R_S [Ω]	C_L [pF]
0.8	179.6	14.296	0.996	0 ~ 50	0.1 ~ 1
1.0	143.7	13.851	1.035		
1.6	89.8	12.913	1.151		
2.0	71.8	12.468	1.228		
5.0	28.7	10.645	1.808		
10.0	14.4	9.276	2.776		



Step Responses



Unlike the 3-pole approximation,

TWA is accurate for inductance-dominant lines.

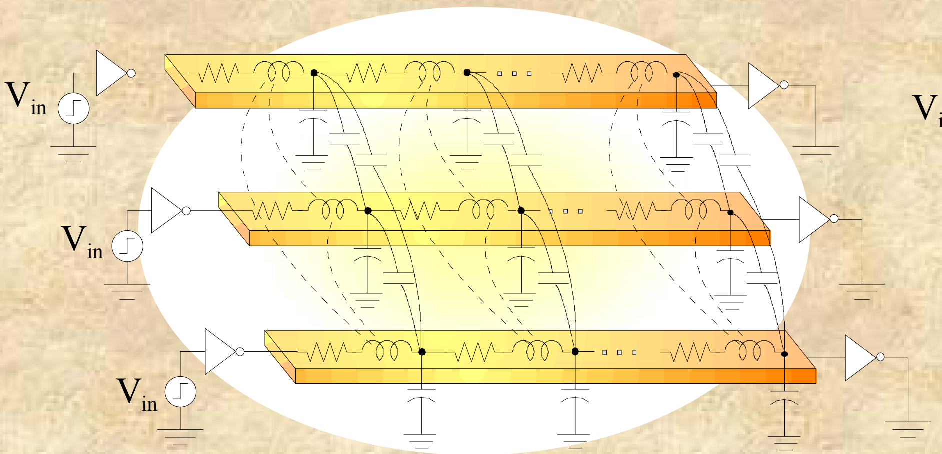
Application of TWA to Multi-Coupled Lines

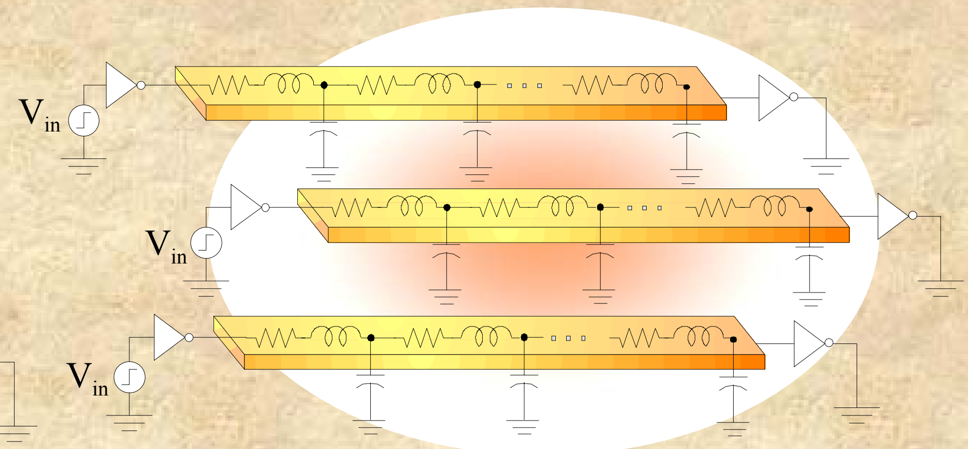
Too Complicated! !

Decoupling

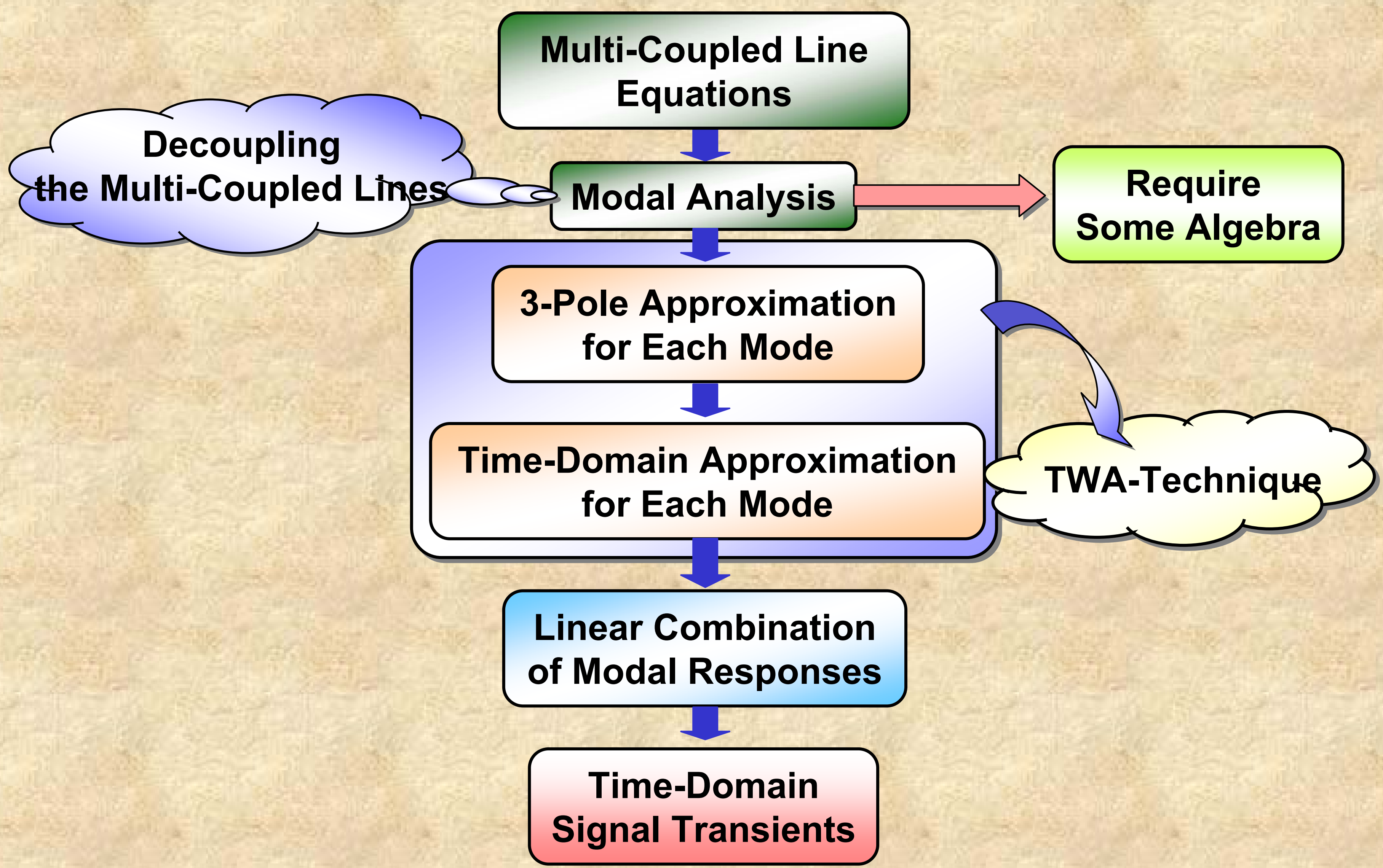
Simple Isolated Lines

➨ TWA-Tech.





TWA-Based Multi-Coupled Line Transient Analysis



Verification of TWA for Multi-Coupled Lines

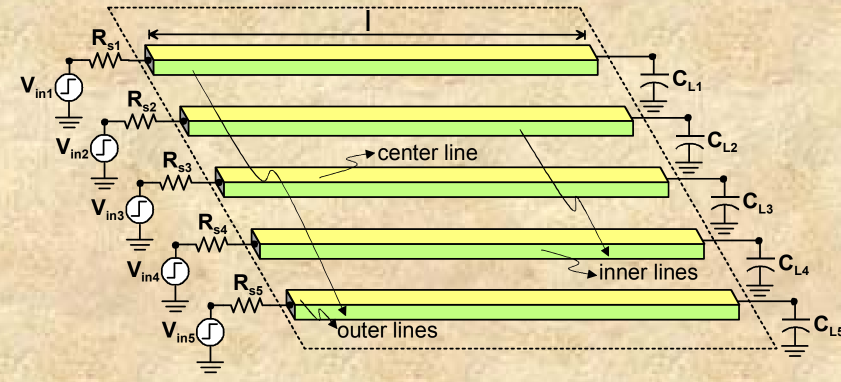
TL Parameters Example for 5 Lines

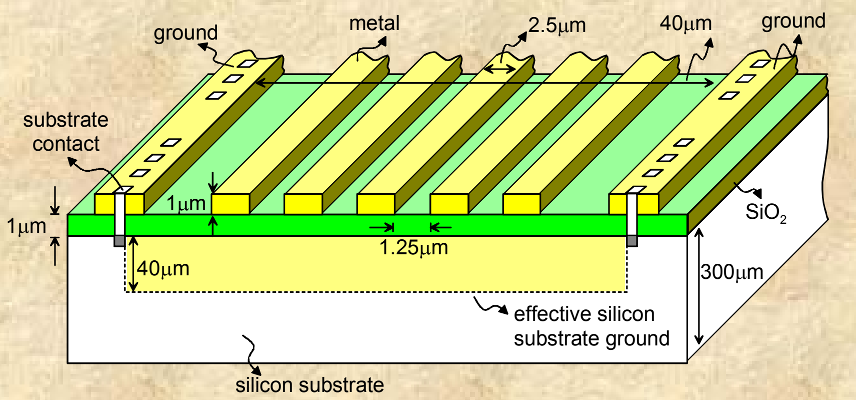
$$[C]=\begin{bmatrix} 2.227 & -0.522 & -0.036 & -0.016 & -0.010 \\ -0.522 & 2.432 & -0.514 & -0.032 & -0.016 \\ -0.036 & -0.514 & 1.327 & -0.514 & -0.036 \\ -0.016 & -0.032 & -0.514 & 2.432 & -0.522 \\ -0.010 & -0.016 & -0.036 & -0.522 & 2.227 \end{bmatrix} [pF/cm]$$

$$[L]=\begin{bmatrix} 7.470 & 5.220 & 4.074 & 3.357 & 2.839 \\ 5.220 & 7.257 & 5.115 & 4.028 & 3.357 \\ 4.074 & 5.115 & 7.214 & 5.115 & 4.074 \\ 3.357 & 4.028 & 5.115 & 7.257 & 5.220 \\ 2.839 & 3.357 & 4.074 & 5.220 & 7.470 \end{bmatrix} [nH/cm]$$

$$[R]=diag(68.966)\ [\Omega/cm]$$

Structure



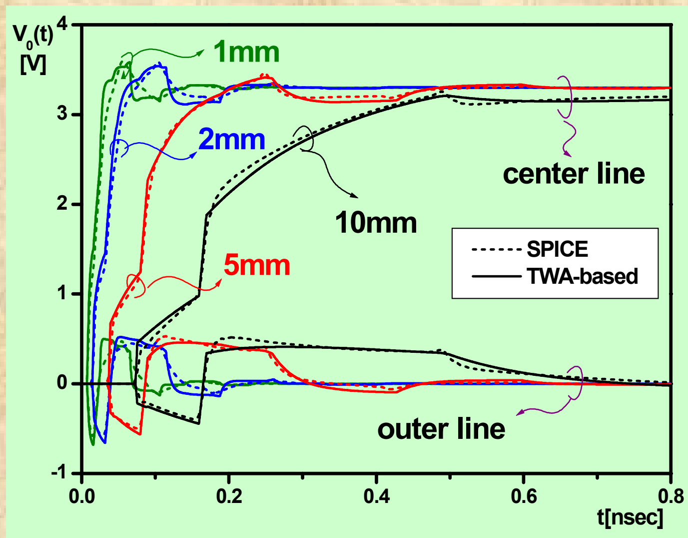


Signal Transients and Crosstalk

Line Length

0↑0

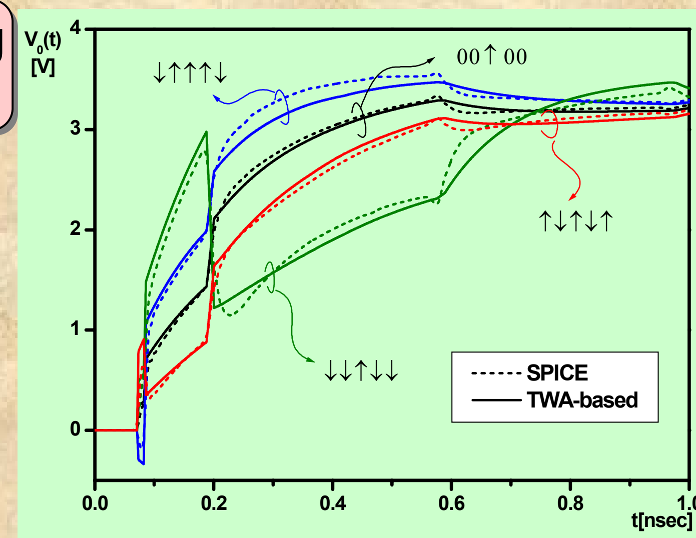
50Ω / 0.1pF



Switching Patterns

10mm

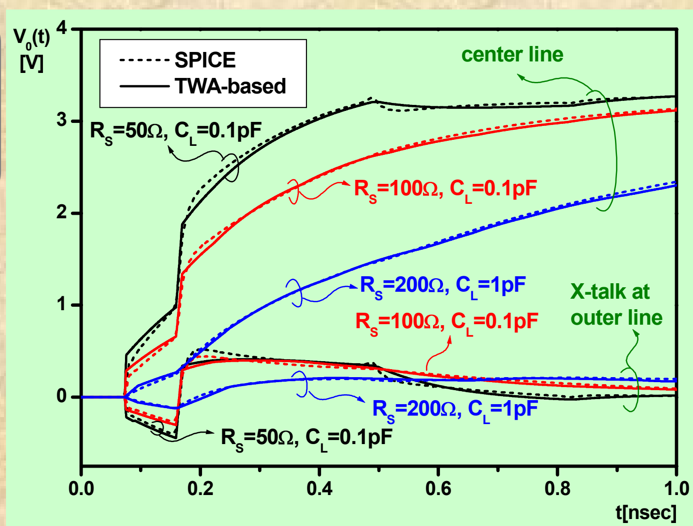
50Ω / 0.1pF



Source/ Load Size

10mm

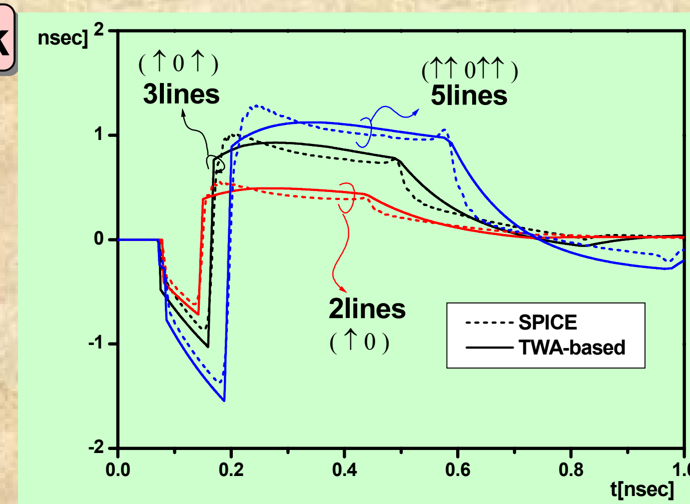
0↑0



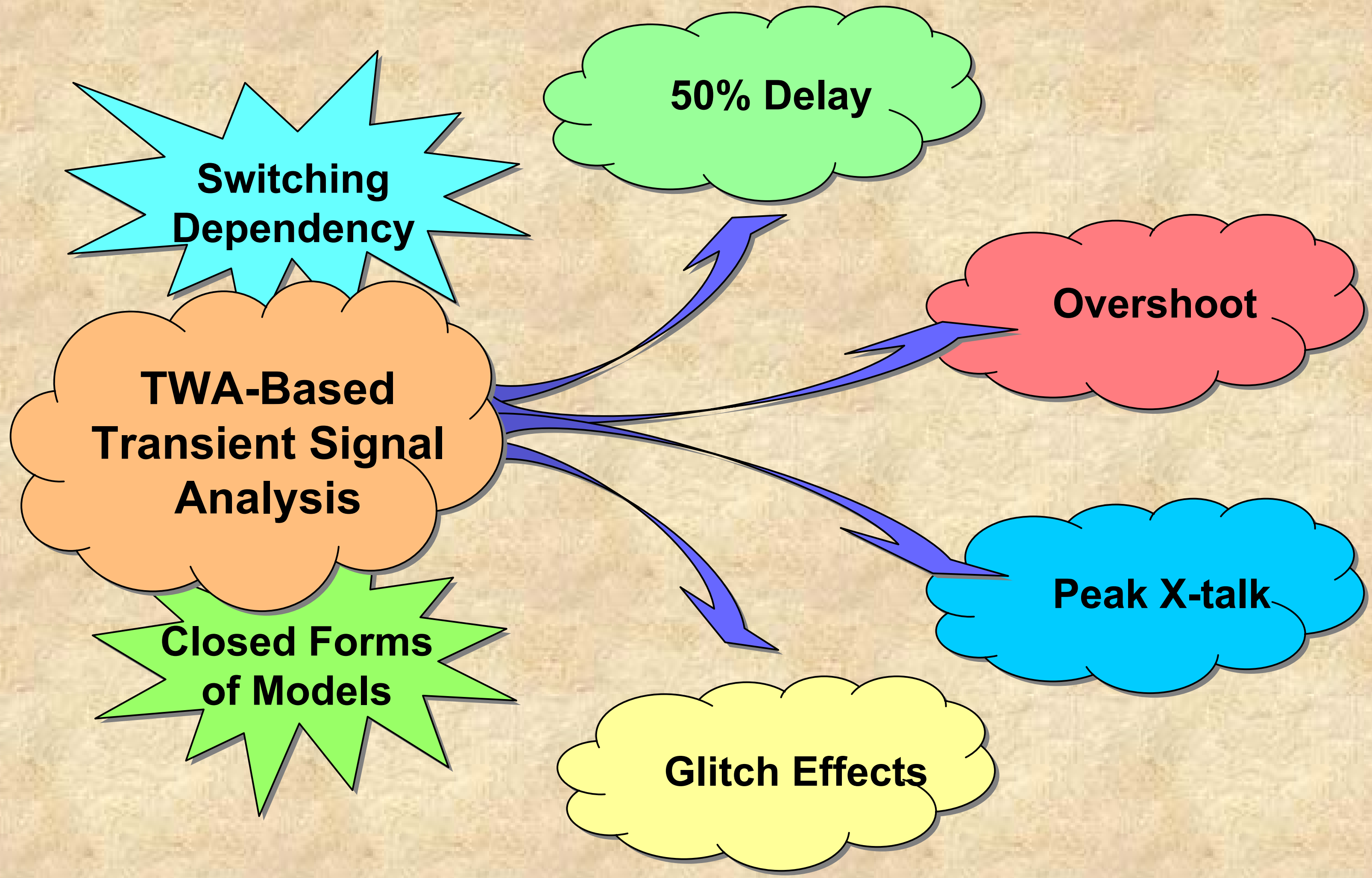
Crosstalk

10mm

50Ω / 0.1pF



TWA-Based Analytical Signal Integrity Models



Frequency-Domain Response

Incident Wave $\left[W(x,\omega)\right]=[S][E][B]$

$[S]$ **: Voltage Eigenmatrix**

$$[E]=\begin{bmatrix} e^{-\gamma_1 x} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{-\gamma_n x} \end{bmatrix}$$

$$\left[W(x=0,\omega)\right]=\left[v_{s1}\cdots v_{sn}\right]^T$$

$$[B]=[S]^{-1}\left[W(x=0,\omega)\right]=\left[b_1,b_2,\cdots,b_n\right]^T$$

$$b_k=\sum_{j=1}^{n} s'_{kj} v_{sj}$$

$$[S]^{-1} \equiv \begin{bmatrix} s'_{11} & s'_{12} & \cdots & s'_{1n} \\ s'_{21} & s'_{22} & \cdots & s'_{2n} \\ \vdots & \vdots & & \vdots \\ s'_{n1} & s'_{n2} & \cdots & s'_{nn} \end{bmatrix}$$

$$W_i(x,\omega)=\sum_{k=1}^{n}\left(\frac{Z_{0k}}{R_{Si}+Z_{0k}}\right) s_{ik} e^{-\gamma_k x} b_k$$

$$\Rightarrow \quad V_i(x,\omega)=\sum_{k=1}^{n}\left(\frac{Z_{0k}}{R_{Si}+Z_{0k}}\right) s_{ik} b_k \left(e^{-\gamma_k x}+\Gamma_k e^{\gamma_k (x-2\ell)}\right)$$

TWA-Based Approximation

$$V_i(x,\omega) = \sum_{k=1}^{n} \left(\frac{Z_{0k}}{R_{Si} + Z_{0k}} \right) s_{ik} b_k \left(e^{-\gamma_k x} + \Gamma_k e^{\gamma_k (x-2\ell)} \right)$$

$$V_i(x,\omega) \approx \sum_{k=1}^{n} s_{ik} b_k Q_k(x,\omega)$$

$$v_i(x,t) \approx \sum_{k=1}^{n} s_{ik} b_k q_k(x,t)$$

$$v_i(x,t) \approx \sum_{k=1}^{n} s_{ik} b_k p_k(x,t)$$

TWA for the k-th Mode

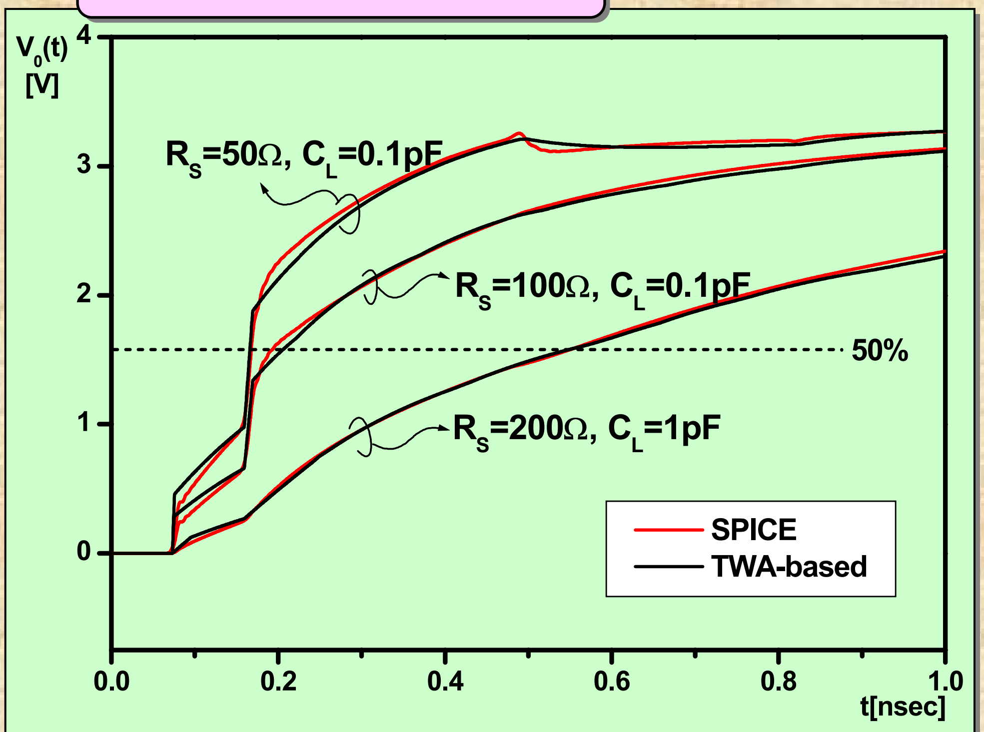
- $Q_k(x,\omega)$ **: 3-pole approximation function**
- $q_k(x,t)$ **: time-domain counter part for 3-pole approximation function**
- $p_k(x,t)$ **: time-domain approximation function**

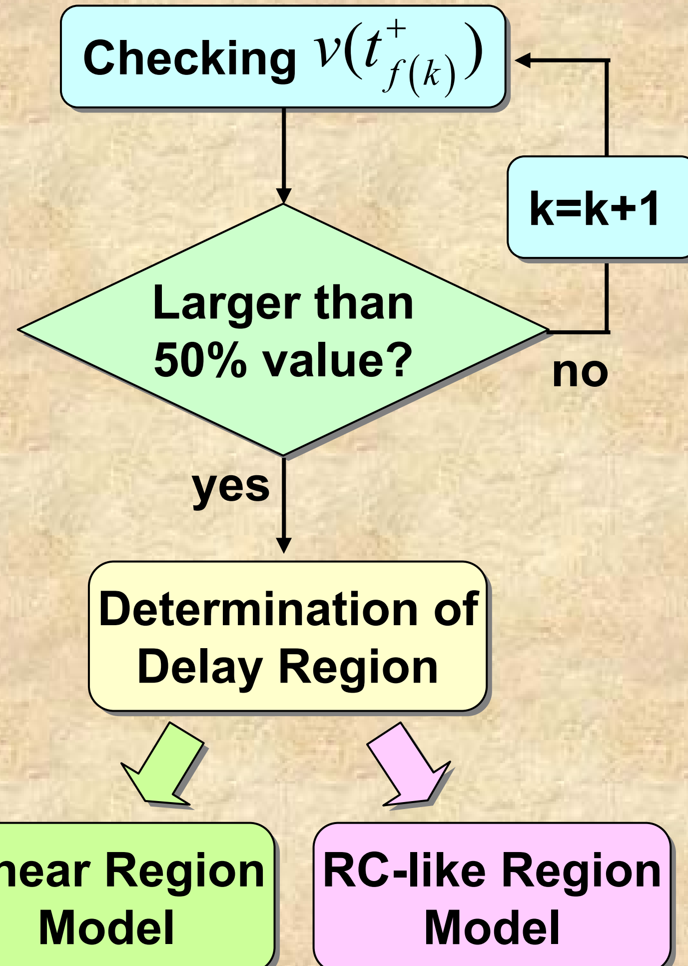
General Waveform

$$v_i(x,t) \approx \sum_{k=1}^{n} \left\{ s_{ik} \left(\sum_{j=1}^{n} s'_{kj} v_{sj} \right) p_k(x,t) \right\}$$

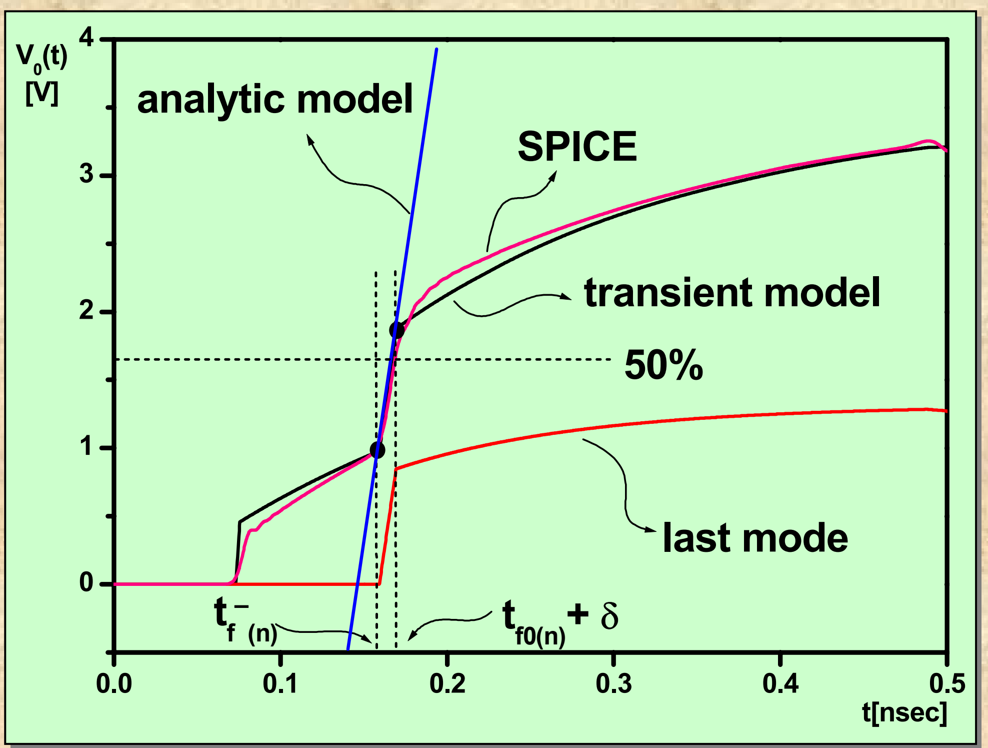
Closed Form of Delay Model

Waveform Variations





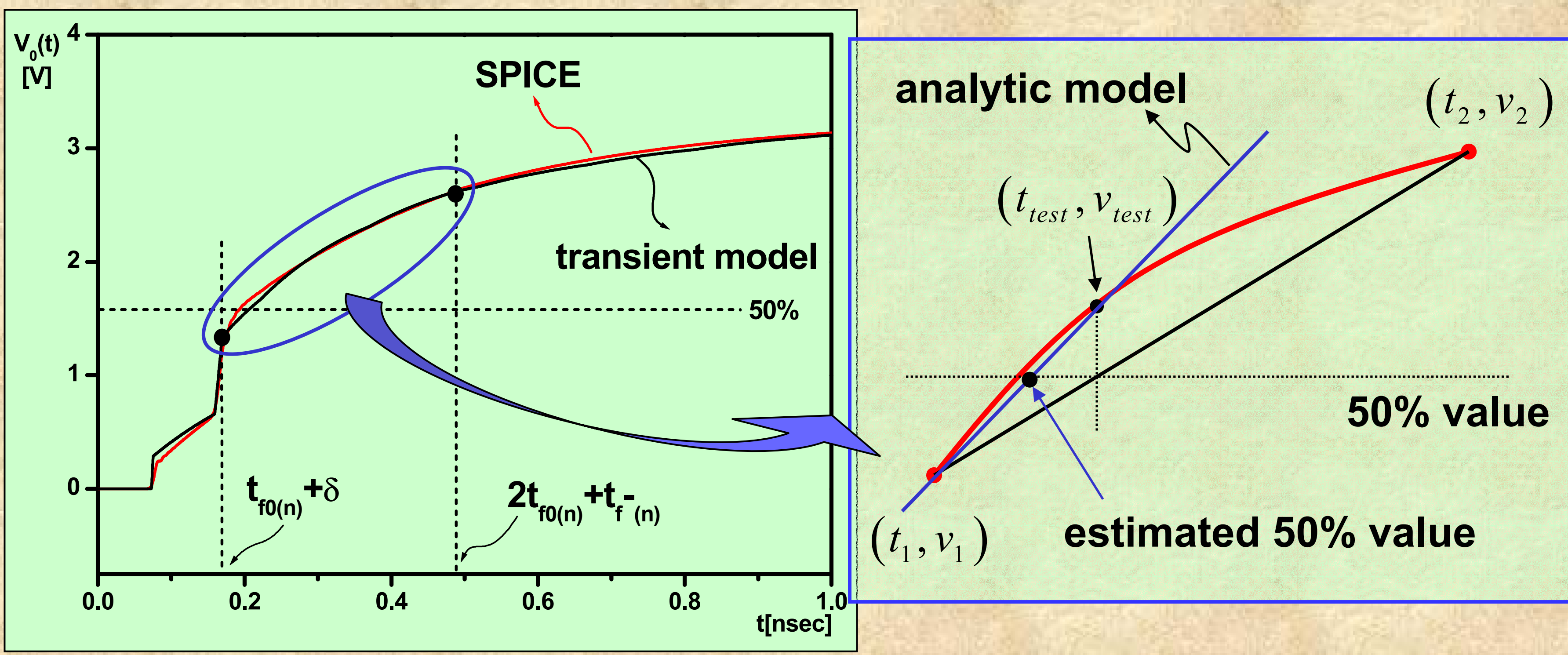
Delay Model for Linear Region



$$v_i(t) \approx \frac{v_i\left(t_{f0(n)} + \delta_{(n)}\right) - v_i\left(t_{f(n)}^{-}\right)}{2\delta_{(n)}}\left(t - t_{f(n)}^{-}\right) + v_i\left(t_{f(n)}^{-}\right)$$

$$t_{50\% delay} = \frac{2\delta_{(n)}}{v_i\left(t_{f0(n)} + \delta_{(n)}\right) - v_i\left(t_{f(n)}^{-}\right)}\left(0.5v_{si} - v_i\left(t_{f(n)}^{-}\right)\right) + t_{f(n)}^{-}$$

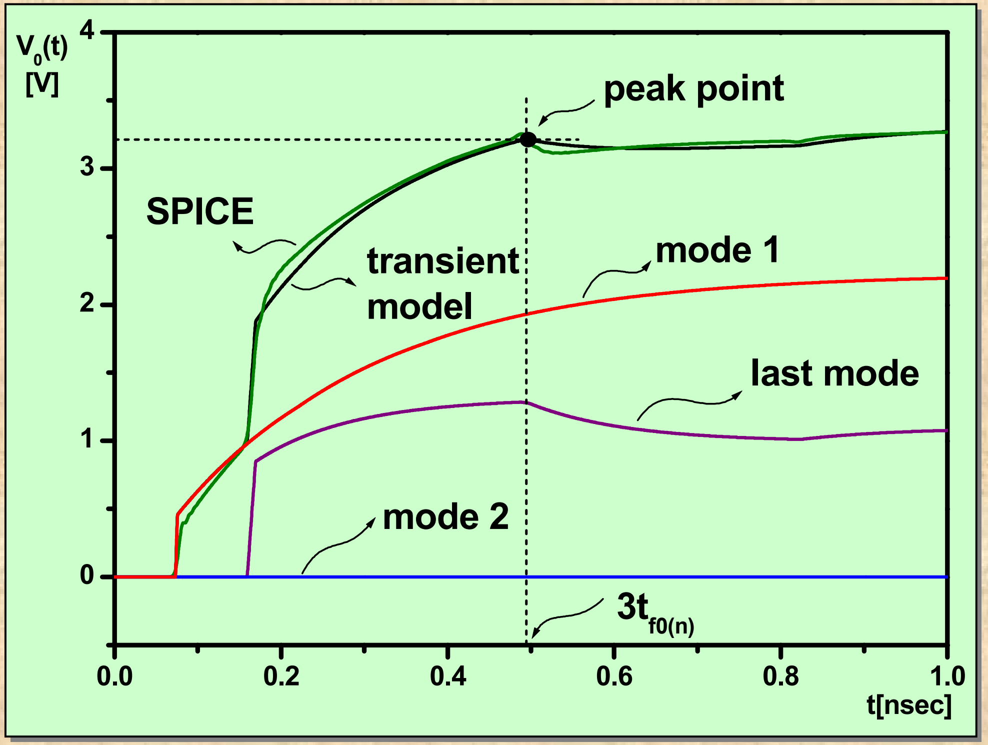
Delay Model for RC-like Region



$$t'_{50\%delay} = \frac{t_{test} - \left(t_{f0(n)} + \delta_{(n)}\right)}{v_{test} - v_i\left(t_{f0(n)} + \delta_{(n)}\right)}\left(0.5v_{si} - v_i\left(t_{f0(n)} + \delta_{(n)}\right)\right) + \left(t_{f0(n)} + \delta_{(n)}\right)$$

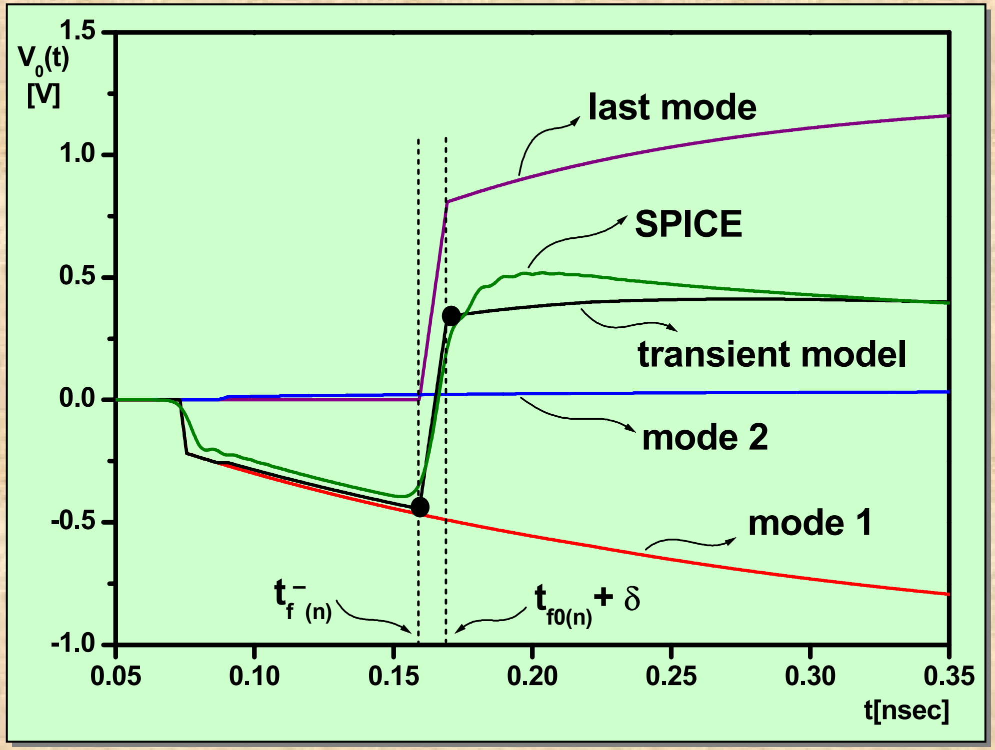
$$v_{test} = v_i\left(t_{test}\right) \qquad t_{test} = \frac{2t^-_{f(n)}}{v_i\left(3t_{f0(n)} - \delta_{(n)}\right) - v_i\left(t_{f0(n)} + \delta_{(n)}\right)}\left(0.5v_{si} - v_i\left(t_{f0(n)} + \delta_{(n)}\right)\right)$$

Closed Form of Overshoot



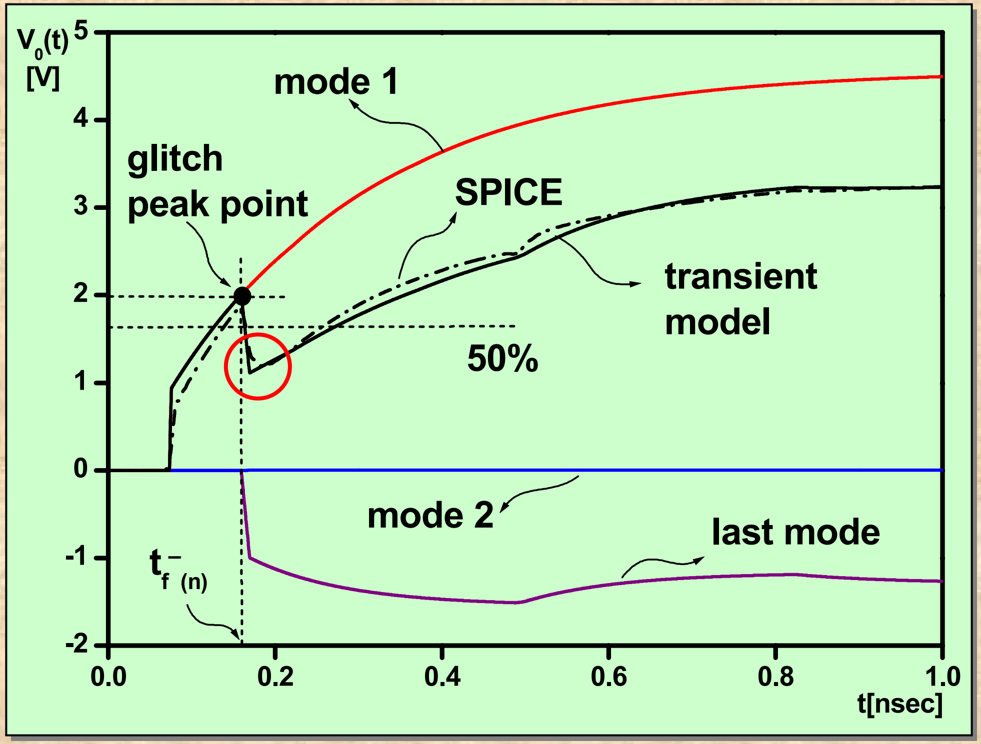
$$v_{i-peak} = MAX\left(v_{si},\ \sum_{k=1}^{n} \left\{ s_{ik} \left(\sum_{j=1}^{n} s'_{kj} \cdot v_{sj} \right) \cdot p_k \left(3t_{f0(n)} \right) \right\} \right)$$

Closed Form of Peak X-talk



$$MAX\left(\left|\sum_{k=1}^{n}\left\{s_{ik}\left(\sum_{j=1}^{n}s'_{kj}\cdot v_{sj}\right)\cdot T_k\left(t^-_{f(n)}\right)\right\}\right|,\ \left|\sum_{k=1}^{n}\left\{s_{ik}\left(\sum_{j=1}^{n}s'_{kj}\cdot v_{sj}\right)\cdot T_k\left(t_{f0(n)}+\delta_{(n)}\right)\right\}\right|\right)$$

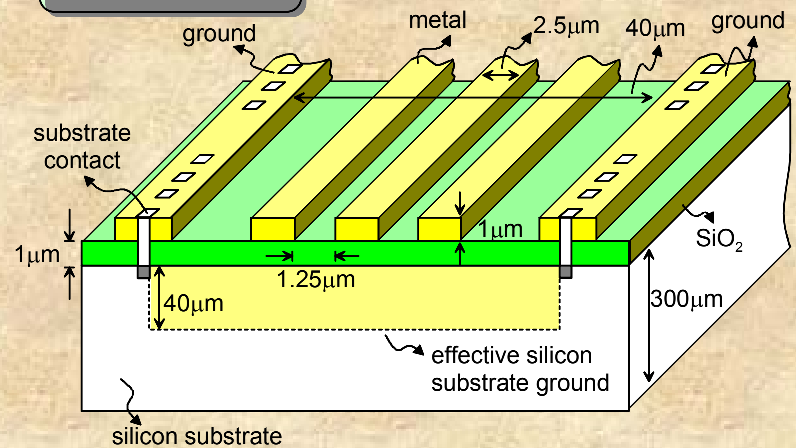
Closed Form of Glitch Signal



$$v_{i-glitch-peak} = v_i\left(t_{f(n)}^{-}\right) \approx \sum_{k=1}^{n}\left\{ s_{ik}\left(\sum_{j=1}^{n} s'_{kj} v_{sj}\right) p_k\left(t_{f(n)}^{-}\right)\right\}$$

Verification of Analytical Signal Integrity Models

Structure



Parameters

$$[L] = \begin{bmatrix} 7.160 & 4.924 & 3.838 \\ 4.924 & 7.010 & 4.924 \\ 3.838 & 4.924 & 7.160 \end{bmatrix} \quad [nH/cm]$$

$$[C] = \begin{bmatrix} 2.225 & -0.522 & -0.042 \\ -0.522 & 2.427 & -0.522 \\ -0.042 & -0.522 & 2.225 \end{bmatrix} \quad [pF/cm]$$

$$[R] = diag(68.966) \quad [\Omega/cm]$$

Basic Patterns

Variables / Test Item	Line Length	Switching Pattern	Source & Load	Input
Delay	10mm	**0↑0**	R_S=50W, C_L=0.1pF	3.3V
X-Talk		↑**0**↑		
Overshoot		**0↑0**		
Glitch	10mm	↓↑↓	R_S=50W,C_L=0.1pF	3.3V

Verification Data(1)

Variable

• Line Lengths

0↑0 Switching

Approximately 5% Error

Line Length	Active Line Delay [psec]		Active Line Overshoot [V]		Quiet Line Crosstalk [V]	
	SPICE	TWA	SPICE	TWA	SPICE	TWA
1mm	19.2	17.4	3.5913	3.3854	0.5807	0.6821
2mm	35.5	33.7	3.5868	3.4848	0.5762	0.6592
5mm	85.2	83.6	3.4601	3.4139	0.53	0.5652
10mm	168.8	166.8	3.2548	3.2060	0.5192	0.4446

Variable

• Switching Patterns

Approximately 2% error

Switching Patterns	Active Line Delay [psec]		Active Line Overshoot [V]		Quiet Line Crosstalk [V]	
	SPICE	TWA	SPICE	TWA	SPICE	TWA
0 ↑ 0	168.8	166.8	3.2548	3.2082	0.5192	0.4446
↑ ↑ ↑	167.6	165.7	4.0301	3.9860	—	—
↑ 0 ↑	170	165.9	3.5262	3.5822	1.0153	1.0275

Verification Data(2)

Variables
- **Input Driver**
- **Output Load**

0↑0 Switching

Items (driver/load)	SPICE [psec]	TWA-based [psec]	Error[%]
$50\Omega/0.1pF$	168.8	166.8	1.2
$100\Omega/0.1pF$	203.2	219.5	8
$200\Omega/1pF$	582.2	589.3	1.2

Variables
- **Line Lengths**

Line Length	↑0↑ crosstalk [V]		↑↑0↑↑ crosstalk [V]	
	SPICE	TWA	SPICE	TWA
1mm	1.176	1.5712	1.6705	2.1156
2mm	1.2022	1.5201	1.7357	2.0934
5mm	1.0829	1.3040	1.6241	1.8696
10mm	1.0153	1.0275	1.3867	1.5456

Approximately 15% Error (Overestimation)

Verification Data(3)

Glitch Signal (↓↑↓ Switching)

Variables

- **Line Lengths**

Line Length	Delay [psec]		Glitch Peak [V]	
	SPICE	TWA	SPICE	TWA
1mm	10.7	9.5	2.452	3.0690
2mm	18.9	16.9	2.4743	2.9691
5mm	51	46.4	2.2505	2.5470
10mm	258.4	275.6	1.8435	2.0068

Approximately 15% Error (Overestimation)

Non-Identical Lines

Variables

- **Line Width**
- **Spacing**

model / items	SPICE	TWA-based	Error[%]
50% delay	158.6 psec	155.7 psec	1.8
Overshoot	3.2265 V	3.2855 V	1.8
Crosstalk	0.5014 V	0.5215 V	4.0

Approximately 5% Error

Execution Time Computation

Execution Environment

SPICE	Model
SUN Ultrasparc-10	AMD Athlon 750MHz

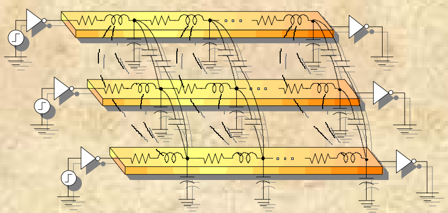
Execution Time

Items / model	3-lines [sec]	5-lines [sec]	Output
SPICE	78	197	Waveform
Eq. (14)	0.03	0.06	50% delay
Eq. (18)	0.03	0.06	Overshoot
Eq. (19)	0.03	0.06	Crosstalk

2500~3000 Times Faster than SPICE !!

Summary of the Presentation

Multiple Lines



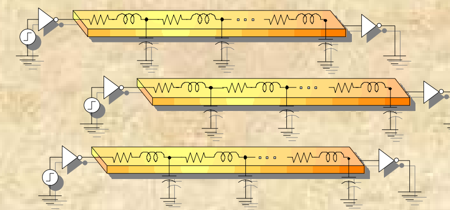
Frequency Response (Telegrapher Equation)

$$\frac{d^2[V(x)]}{dx^2} = [Z][Y][V(x)]$$

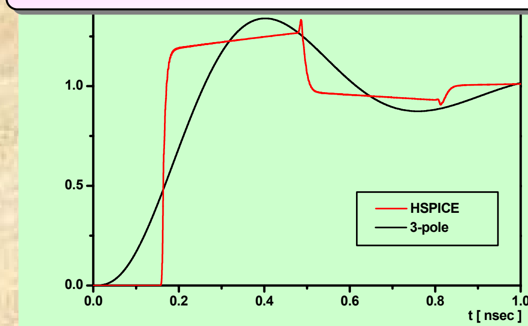
$$[Z] = [R] + s[L]$$

$$[Y] = [G] + s[C]$$

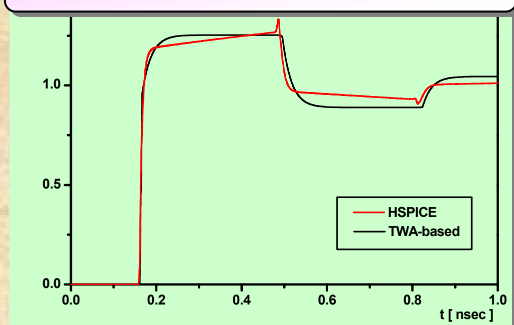
Modal Decoupling



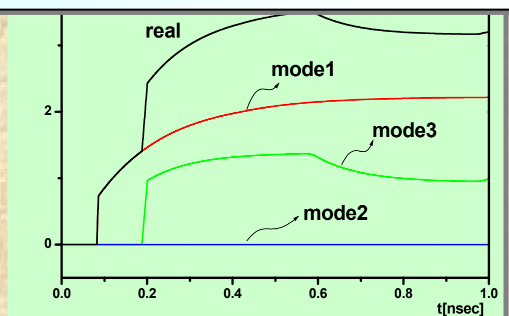
3-Pole Approxi. In Freq.-Domain



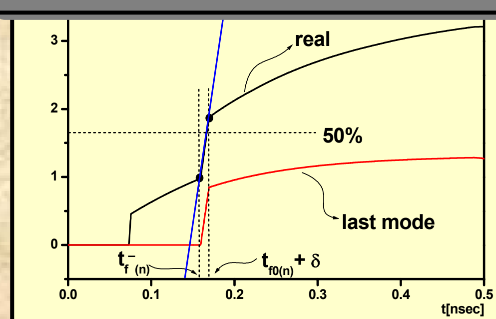
Time-Domain Approximation



Analytical Signal Transient Model



Delay, X-talk, Overshoot, Glitch



Verification

items \ model	SPICE	TWA-based	Error [%]
50% delay	158.6 psec	155.7 psec	1.8
Overshoot	3.2265 V	3.2855 V	1.8
Crosstalk	0.5014 V	0.5215 V	4.0

Conclusion

- New Analytical Signal Integrity Models.
- Excellent Agreement with Approximately 5% Error.
- Considered to be a Good Conservative Estimation.
- 2500 ~ 3000 times Faster than SPICE Simulation.