

# **Analytical Signal Integrity Verification Models for Inductance-Dominant Multi-Coupled VLSI Interconnect**

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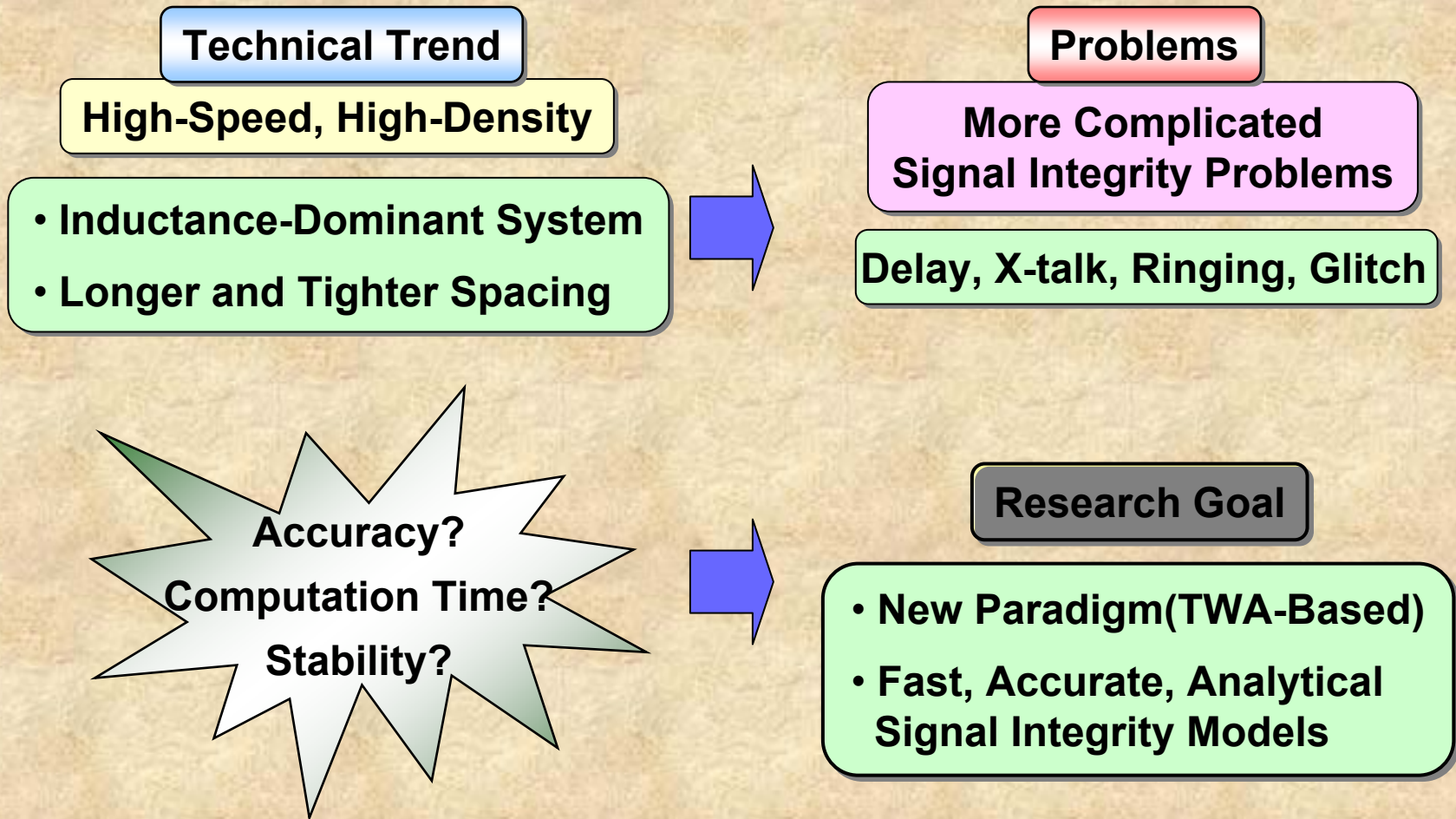
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# Outline

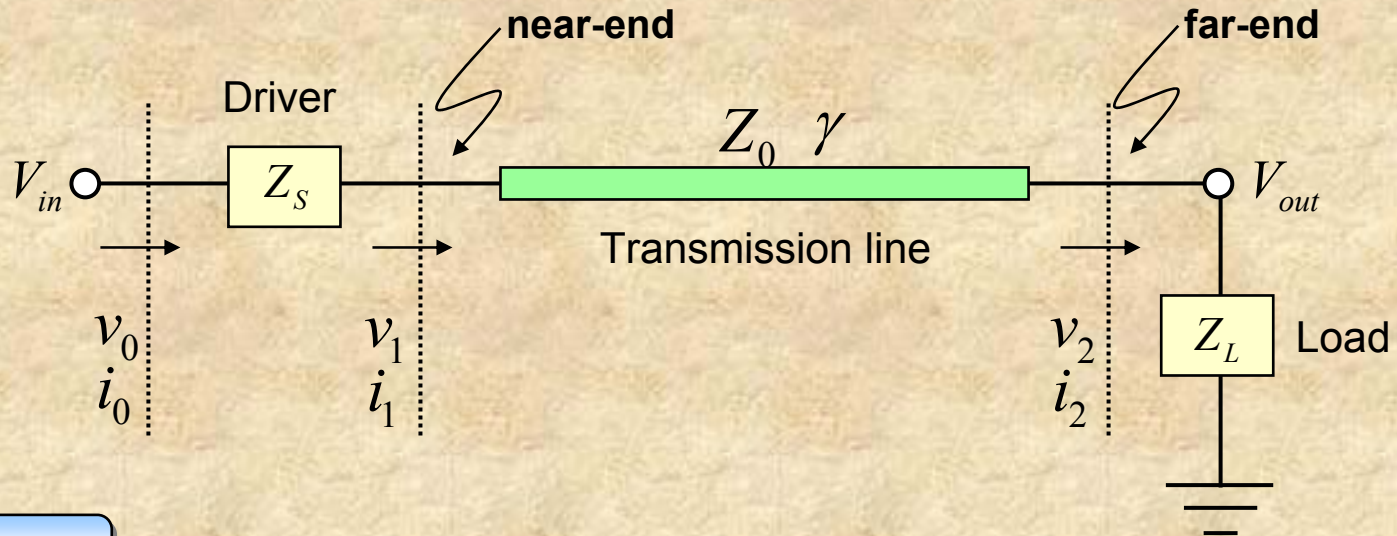
- **Technical Trend and Problems**
- **TWA-Technique**
- **Multi-Coupled Lines**
- **Analytical Models and Verification**
- **Summary and Conclusion**

# Technical Challenge



# Problems

## System Function of a Single Transmission Line



**Far-end**

$$H_{far}(s) = \frac{v_2}{v_0} = \frac{1}{\cosh(\gamma\ell) + \frac{Z_0}{Z_L} \sinh(\gamma\ell) + \frac{Z_S}{Z_0} \sinh(\gamma\ell) + \frac{Z_S}{Z_L} \cosh(\gamma\ell)}$$

**Near-end**

$$H_{near}(s) = \frac{v_1}{v_0} = \frac{Z_0 Z_L \cosh(\gamma\ell) + Z_0^2 \sinh(\gamma\ell)}{Z_0 (Z_S + Z_L) \cosh(\gamma\ell) + (Z_0^2 + Z_S Z_L) \sinh(\gamma\ell)}$$

# Unit Step Response in a Single TL

$$V_{0-far}(s) = \frac{1}{s} \cdot H_{far}(s)$$

$$= \frac{1}{s \cdot \left\{ \cosh(\gamma\ell) + \frac{Z_0}{Z_L} \sinh(\gamma\ell) + \frac{Z_S}{Z_0} \sinh(\gamma\ell) + \frac{Z_S}{Z_L} \cosh(\gamma\ell) \right\}}$$

→

$$v_{0-far}(t) = \mathcal{L}^{-1} \left\{ \frac{1}{s} \cdot H_{far}(s) \right\}$$

$$V_{0-near}(s) = \frac{1}{s} \cdot H_{near}(s)$$

$$= \frac{Z_0 Z_L \cosh(\gamma\ell) + Z_0^2 \sinh(\gamma\ell)}{s \cdot \left\{ Z_0 (Z_S + Z_L) \cosh(\gamma\ell) + (Z_0^2 + Z_S Z_L) \sinh(\gamma\ell) \right\}}$$

→

$$v_{0-near}(t) = \mathcal{L}^{-1} \left\{ \frac{1}{s} \cdot H_{near}(s) \right\}$$

**Require Numerical Integration!!**

# Dominant Pole Approximation(using 3-Dominant Poles)

## Far-end

$$H_{far}(s) = \frac{v_2}{v_0} = \frac{1}{\cosh(\gamma\ell) + \frac{Z_0}{Z_L} \sinh(\gamma\ell) + \frac{Z_S}{Z_0} \sinh(\gamma\ell) + \frac{Z_S}{Z_L} \cosh(\gamma\ell)}$$

$$v_{0-far}(t) = \mathcal{L}^{-1} \left\{ \frac{1}{s} \cdot H_{far}(s) \right\} \approx \mathcal{L}^{-1} \left\{ \frac{1}{s(s-s_1)(s-s_2)(s-s_3)} \right\}$$

$$= \mathcal{L}^{-1} \left\{ \frac{a_0}{s} + \frac{a_1}{s-s_1} + \frac{a_2}{s-s_2} + \frac{a_3}{s-s_3} \right\} \square v_{03-far}(t)$$

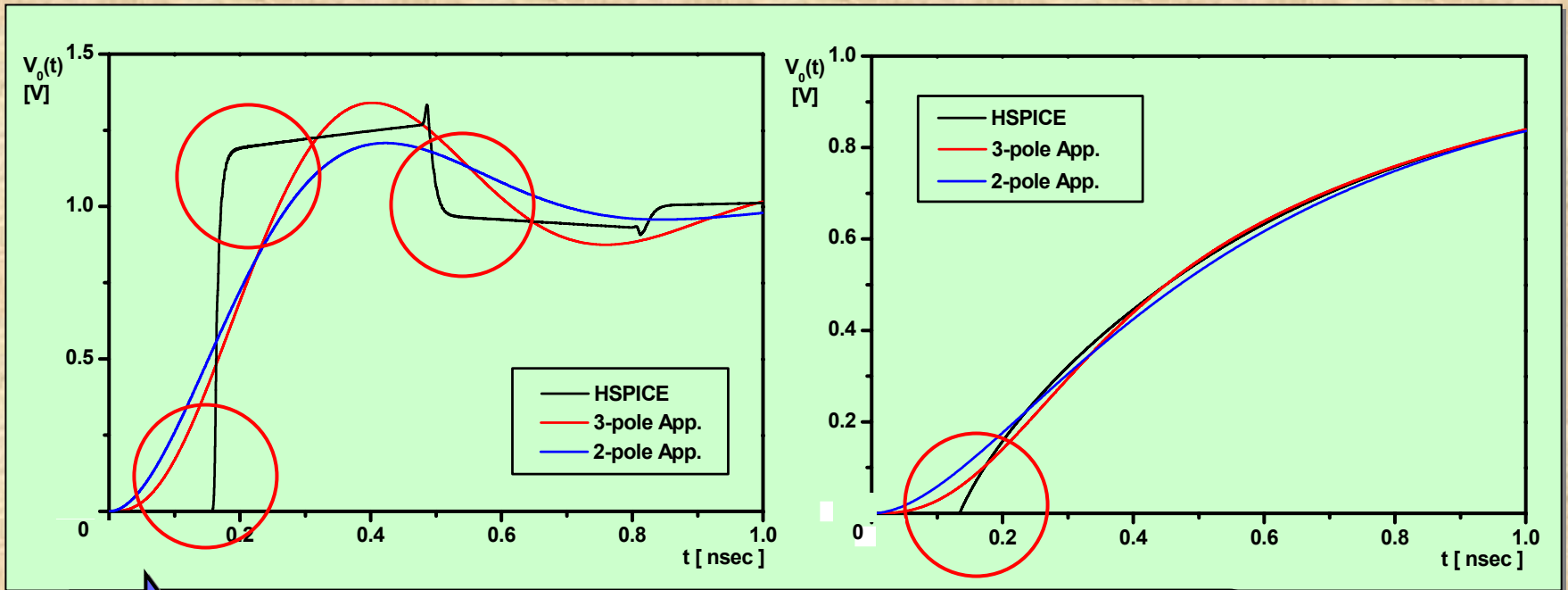
**Do not Require  
Numerical Integration!!**

## Near-end

$$H_{near}(s) = \frac{v_1}{v_0} = \frac{Z_0 Z_L \cosh(\gamma\ell) + Z_0^2 \sinh(\gamma\ell)}{Z_0 (Z_S + Z_L) \cosh(\gamma\ell) + (Z_0^2 + Z_S Z_L) \sinh(\gamma\ell)}$$

$$v_{0-near}(t) = \mathcal{L}^{-1} \left\{ \frac{1}{s} \cdot H_{near}(s) \right\} \approx \mathcal{L}^{-1} \left\{ \frac{1}{s} \cdot \frac{q_1 + q_2 s + q_3 s^2 + q_4 s^3}{p_1 + p_2 s + p_3 s^2 + p_4 s^3} \right\} \square v_{03-near}(t)$$

# Problems of 3-Pole Approximation



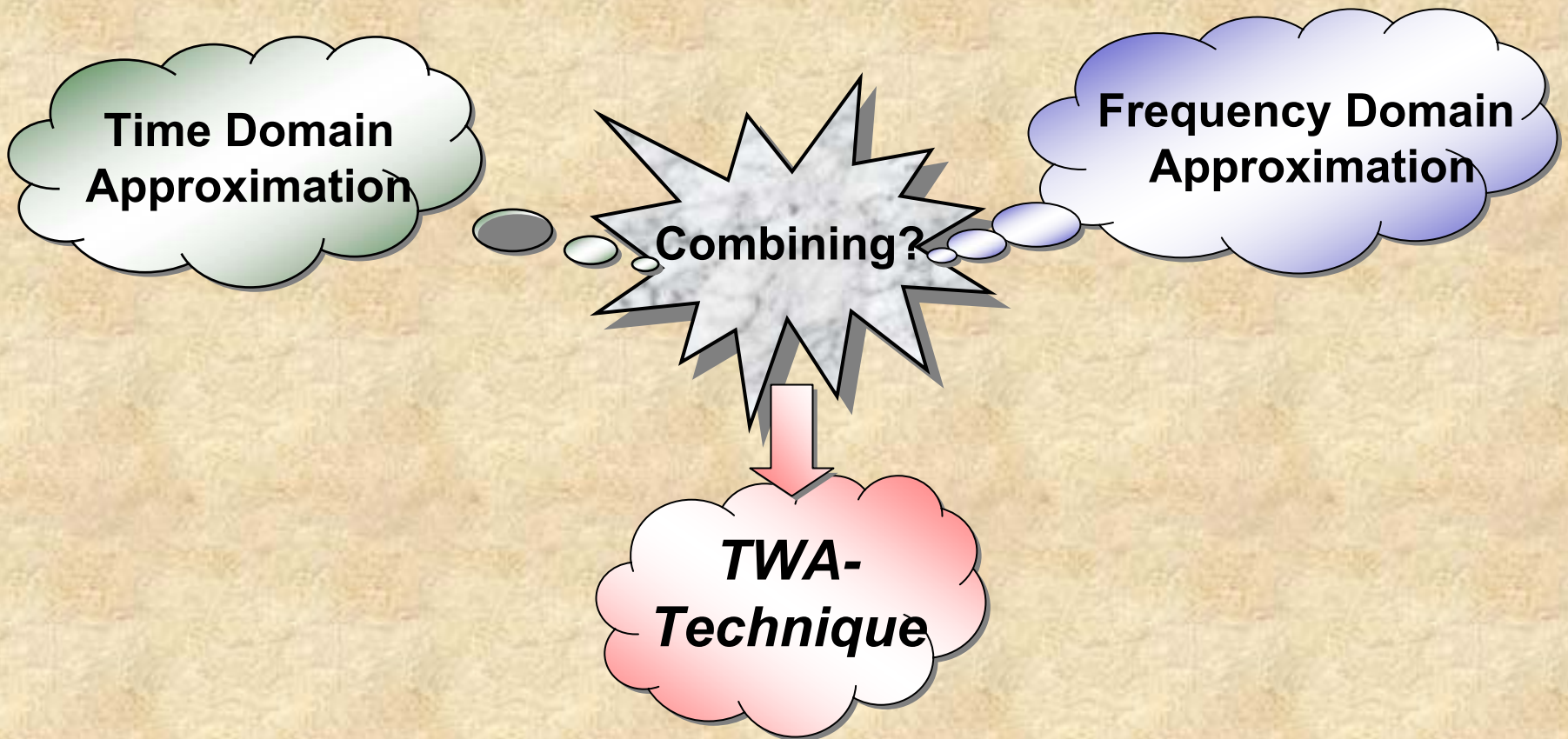
**Dominant pole approximation**

**does not reflect “Inductance-Dominant” effects.**

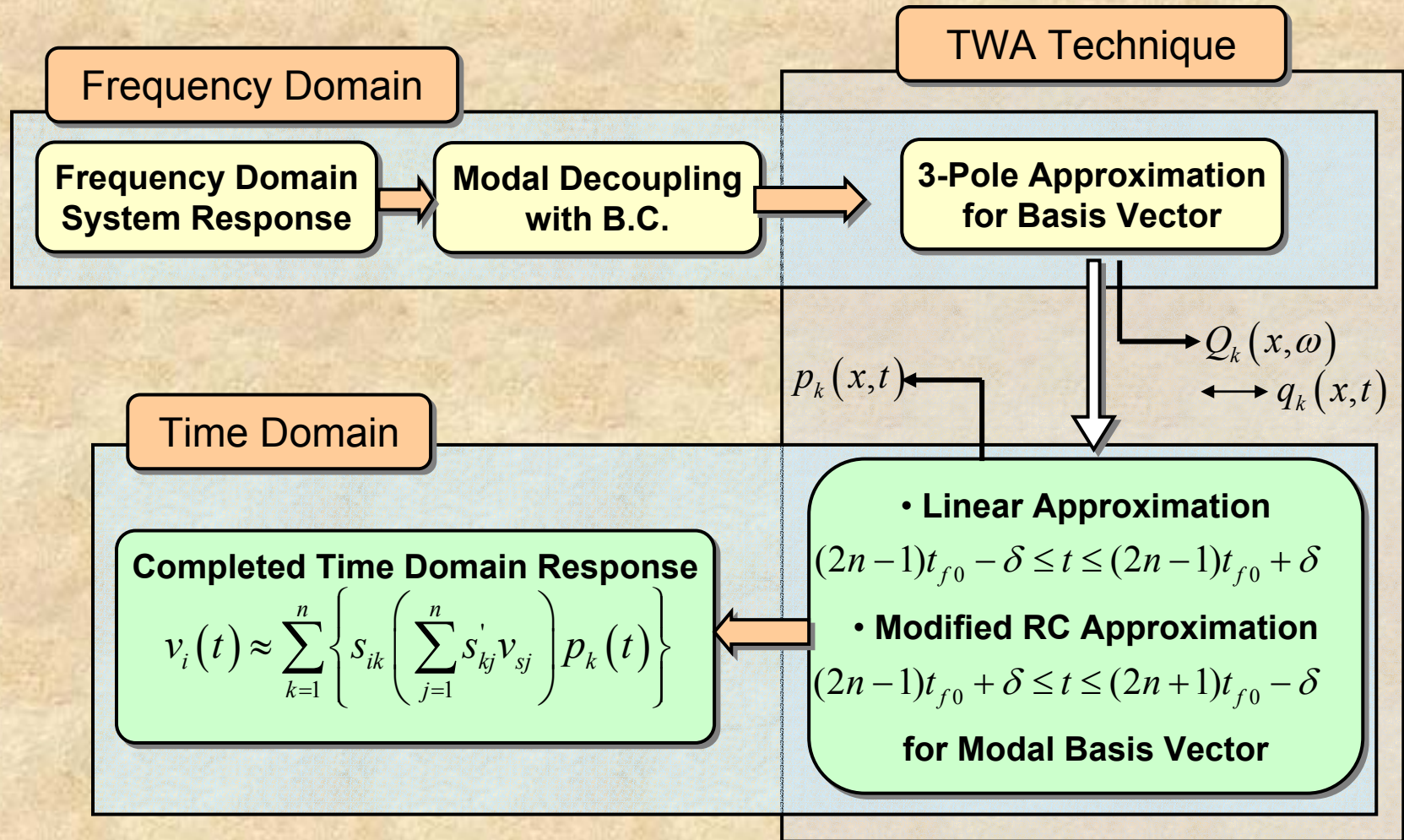
***How to Incorporate  
the High-Frequency Effect?***

# Traveling-Wave-Based Waveform Approximation(TWA)

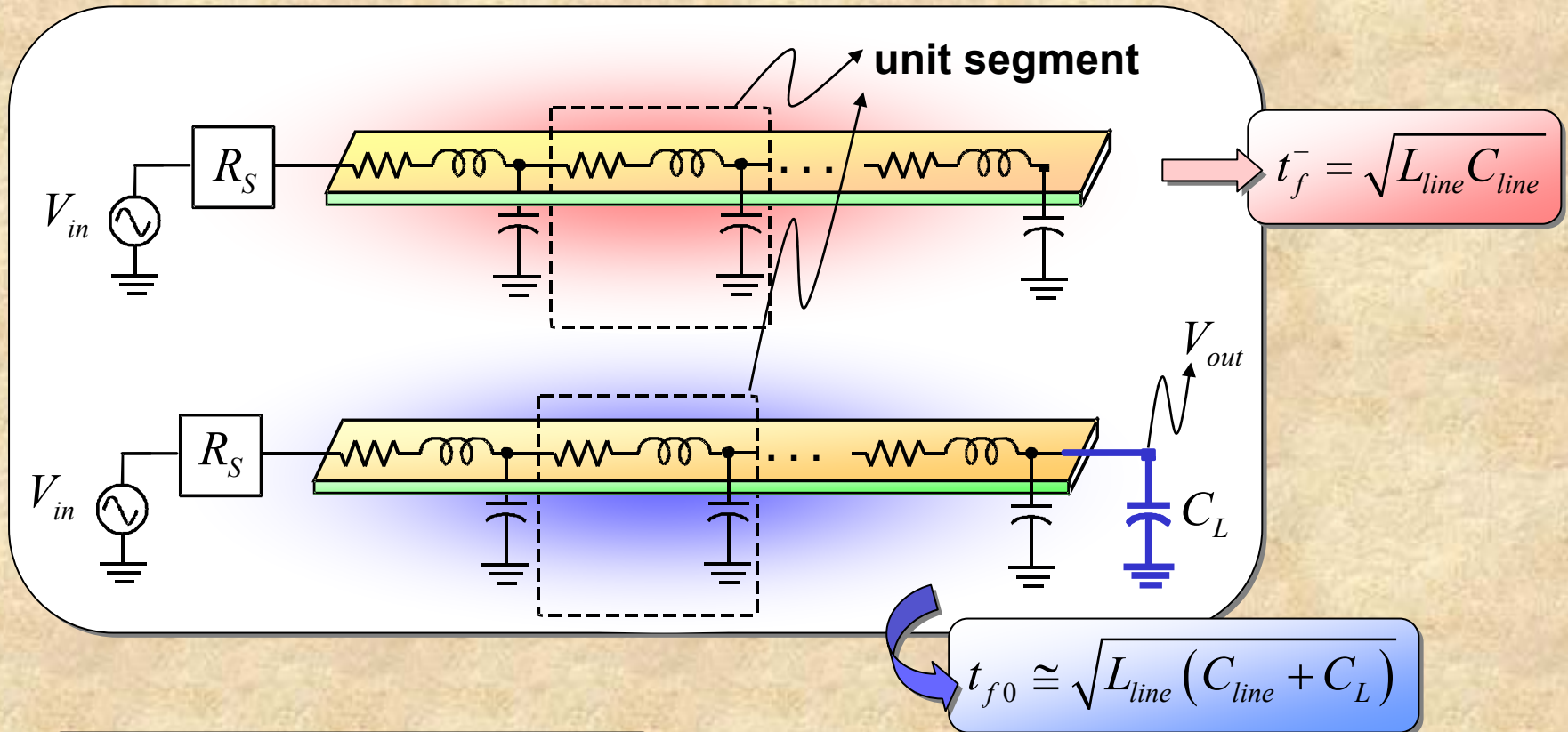
[Y. Eo, et al., "Traveling-wave-based ~," will be published in IEEE T-CAD ]



# TWA-Based Time-Domain Transient Signal Characterizations



# Effective Time of Flight : Loading Effect

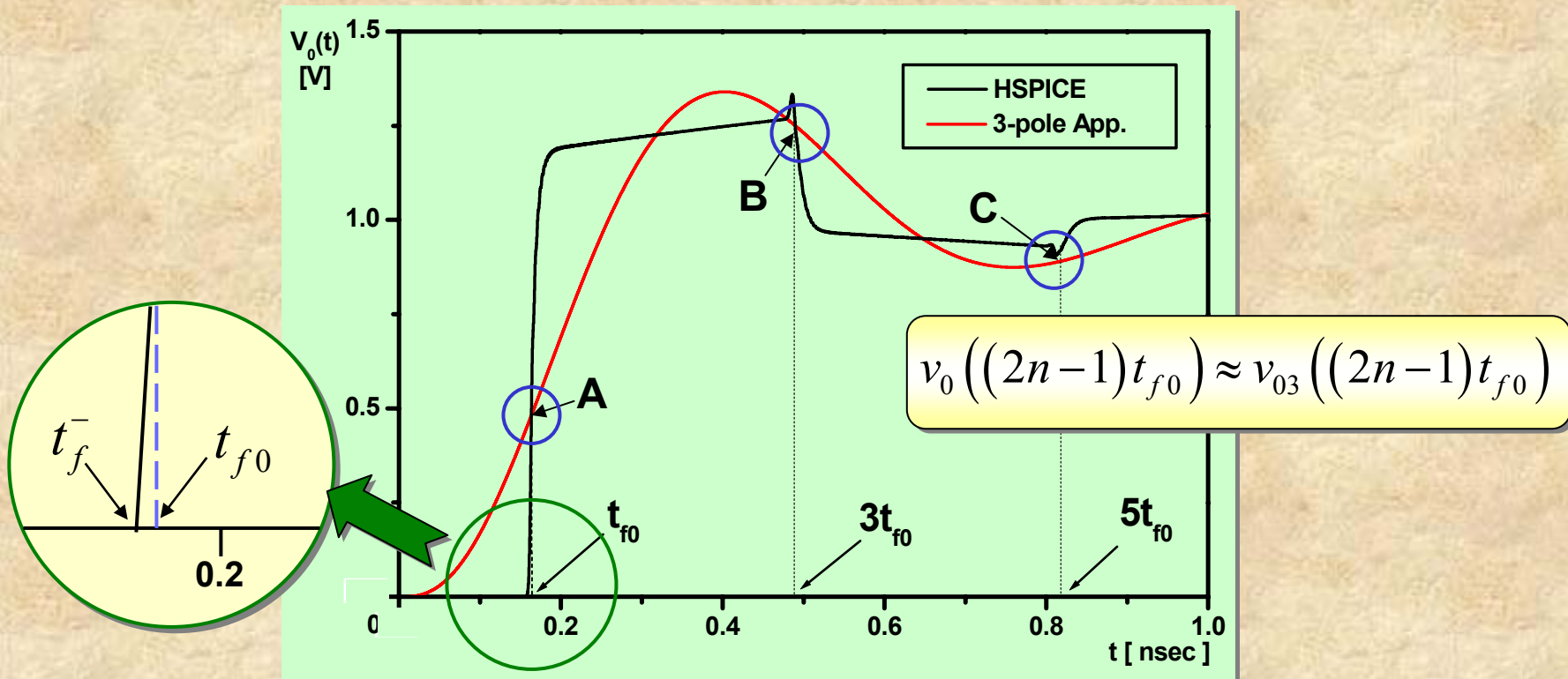


Capacitance-Loading Effect

Increase the effective time of flight ( $t_{f0}$ )

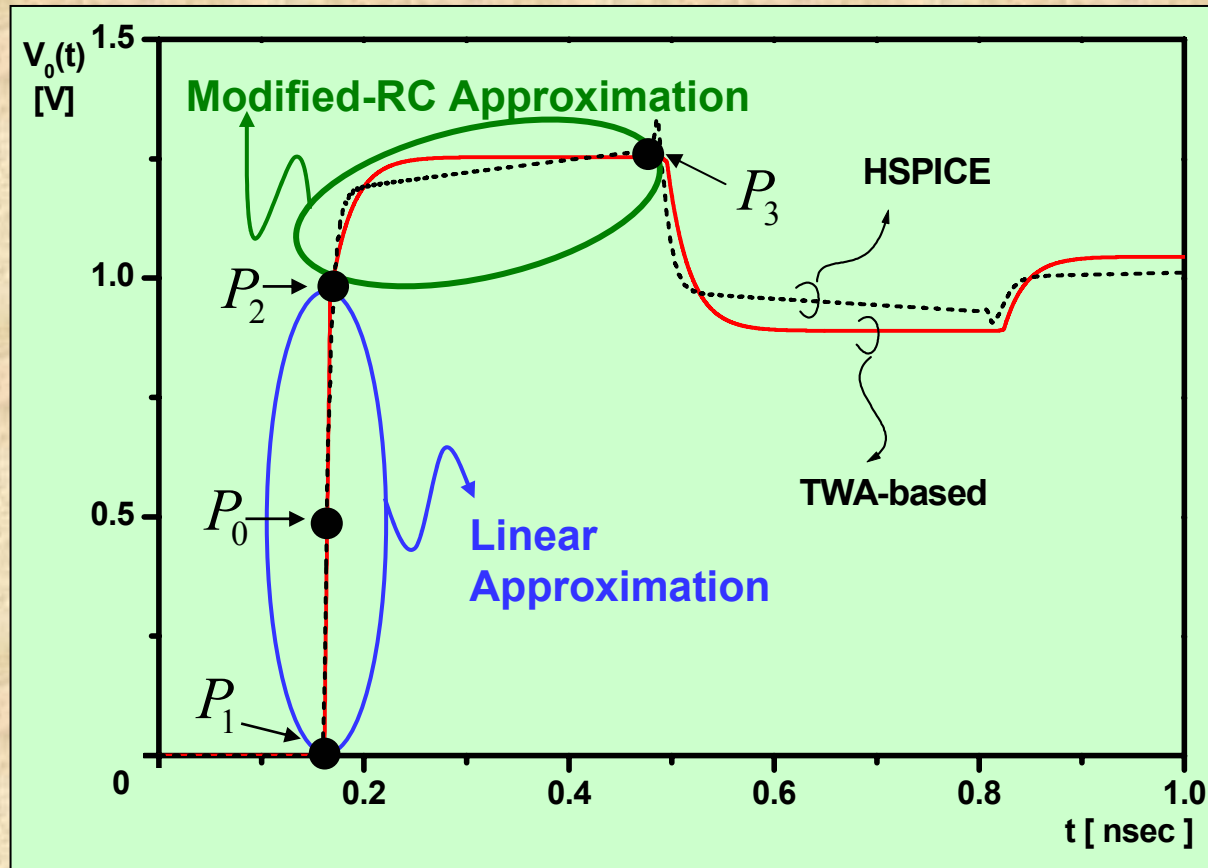
# Frequency-Domain Characteristics (Low-Frequency Characteristics)

➔ with 3-Pole-Based Frequency Domain Response



➔ **“Reflection” means “fast transient”.**

# Time-Domain Characteristics (High-Frequency Characteristics)



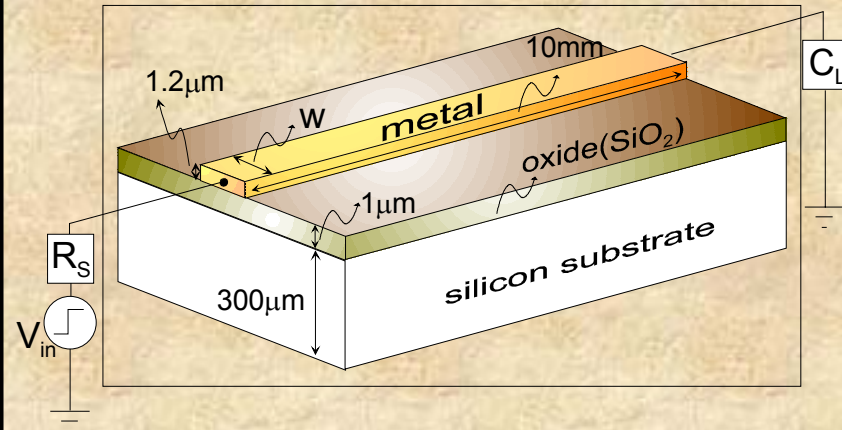
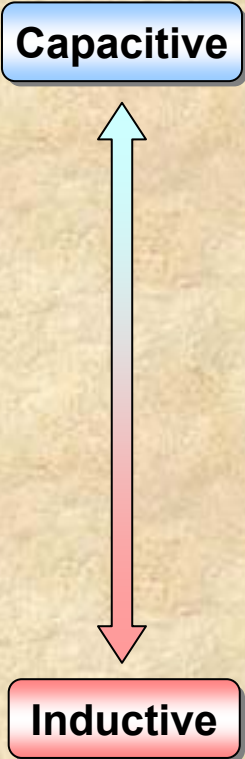
*We can determine the analytical form of expressions*

*Since we know two points.*

# Verification of TWA in a Single Line

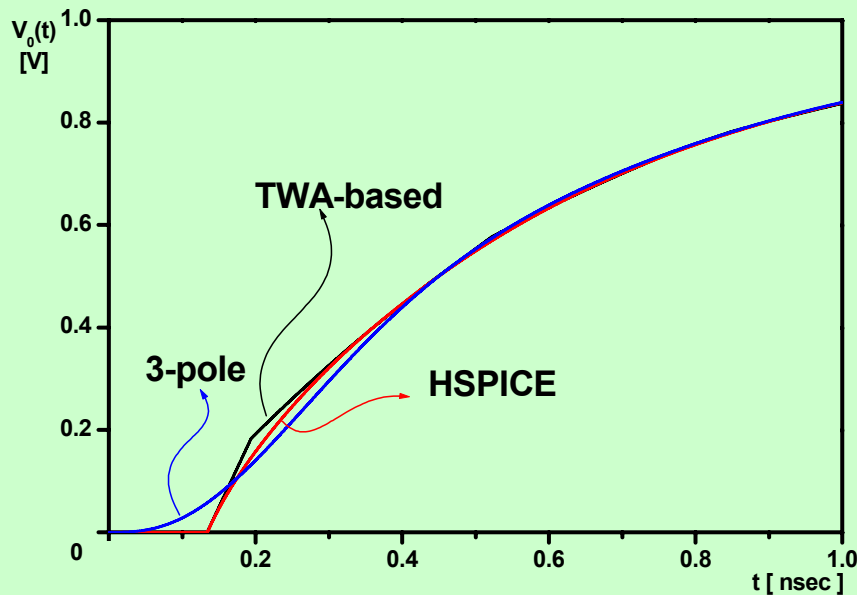
**TL Parameters**

Width [ $\mu\text{m}$ ]	R [ $\Omega/\text{cm}$ ]	L [nH/cm]	C [pF/cm]	$R_S$ [ $\Omega$ ]	$C_L$ [pF]
0.8	179.6	14.296	0.996	0 ~ 50	0.1 ~ 1
1.0	143.7	13.851	1.035		
1.6	89.8	12.913	1.151		
2.0	71.8	12.468	1.228		
5.0	28.7	10.645	1.808		
10.0	14.4	9.276	2.776		

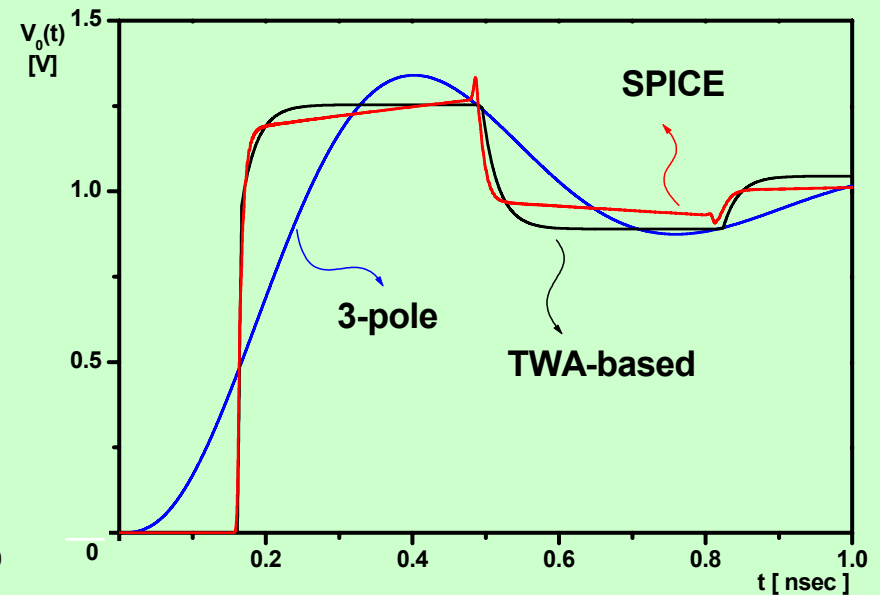


# Step Responses

## Capacitance-Dominant



## Inductance-Dominant



*Unlike the 3-pole approximation,*

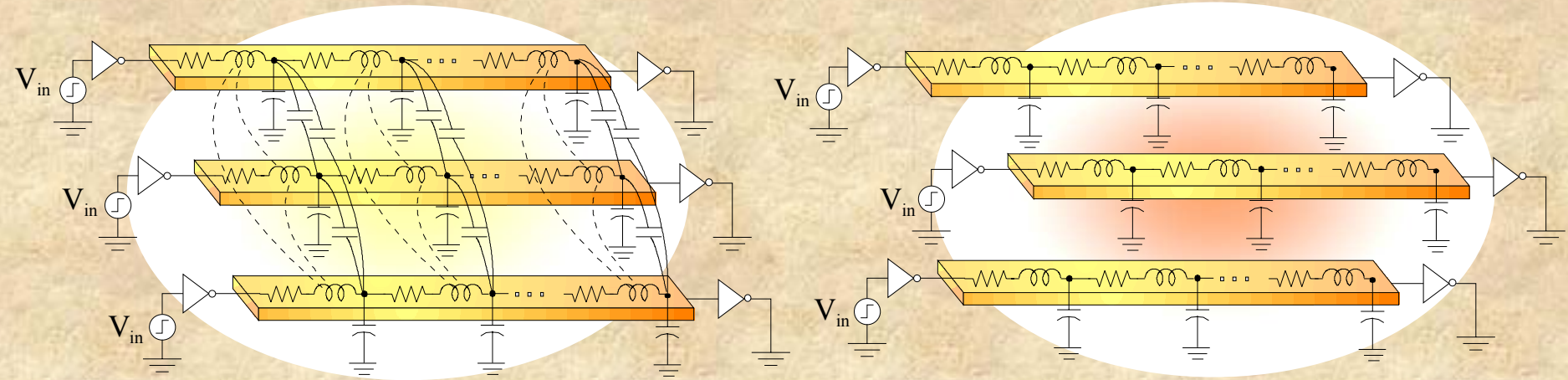
*TWA is accurate for inductance-dominant lines.*

# Application of TWA to Multi-Coupled Lines

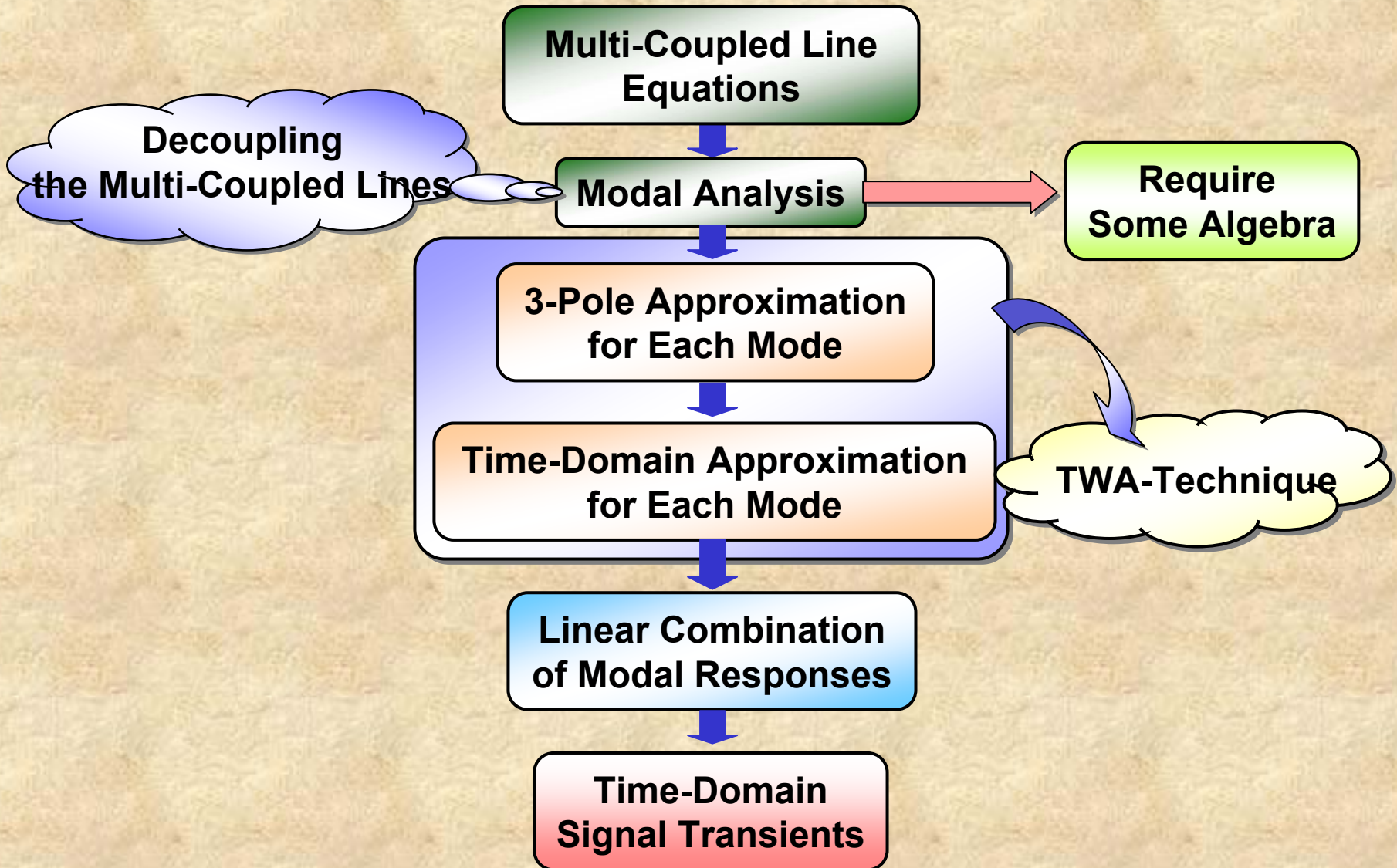
Too  
Complicated!  
!

*Decoupling*

Simple  
Isolated Lines  
→ TWA-Tech.



# TWA-Based Multi-Coupled Line Transient Analysis



# Verification of TWA for Multi-Coupled Lines

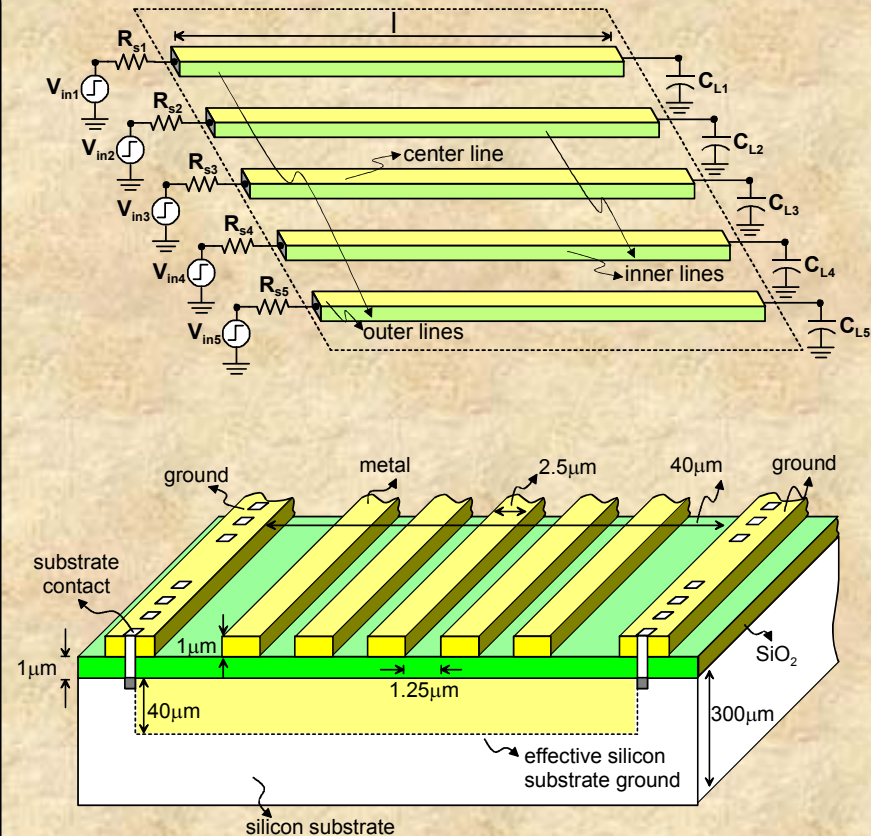
## TL Parameters Example for 5 Lines

$$[C] = \begin{bmatrix} 2.227 & -0.522 & -0.036 & -0.016 & -0.010 \\ -0.522 & 2.432 & -0.514 & -0.032 & -0.016 \\ -0.036 & -0.514 & 1.327 & -0.514 & -0.036 \\ -0.016 & -0.032 & -0.514 & 2.432 & -0.522 \\ -0.010 & -0.016 & -0.036 & -0.522 & 2.227 \end{bmatrix} [pF/cm]$$

$$[L] = \begin{bmatrix} 7.470 & 5.220 & 4.074 & 3.357 & 2.839 \\ 5.220 & 7.257 & 5.115 & 4.028 & 3.357 \\ 4.074 & 5.115 & 7.214 & 5.115 & 4.074 \\ 3.357 & 4.028 & 5.115 & 7.257 & 5.220 \\ 2.839 & 3.357 & 4.074 & 5.220 & 7.470 \end{bmatrix} [nH/cm]$$

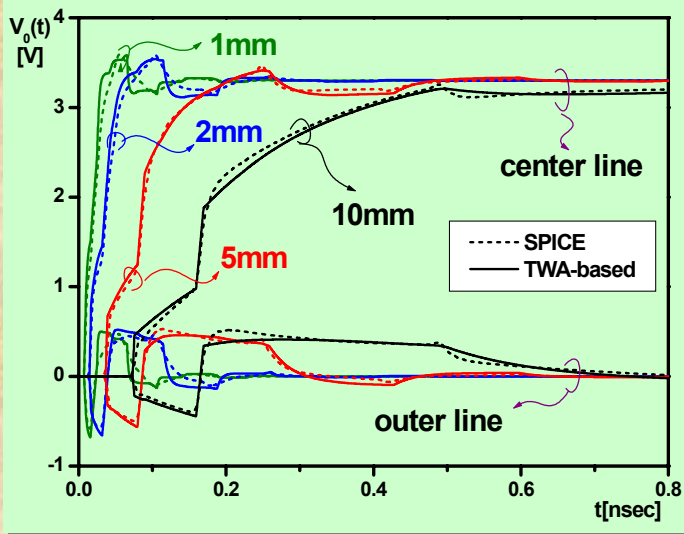
$$[R] = \text{diag}(68.966) [\Omega/cm]$$

## Structure



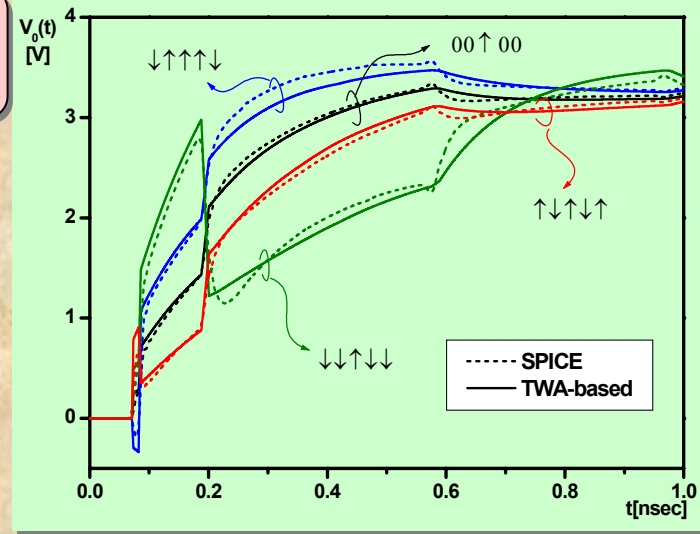
# Signal Transients and Crosstalk

Line Length



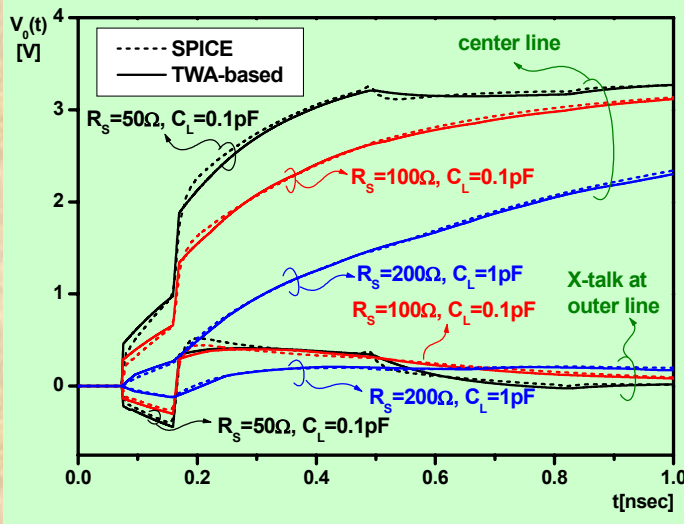
0↑0  
50Ω / 0.1pF

Switching Patterns



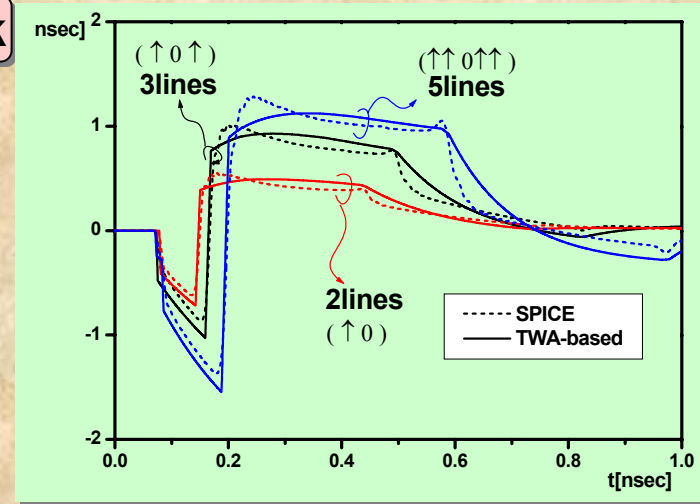
10mm  
50Ω / 0.1pF

Source/ Load Size



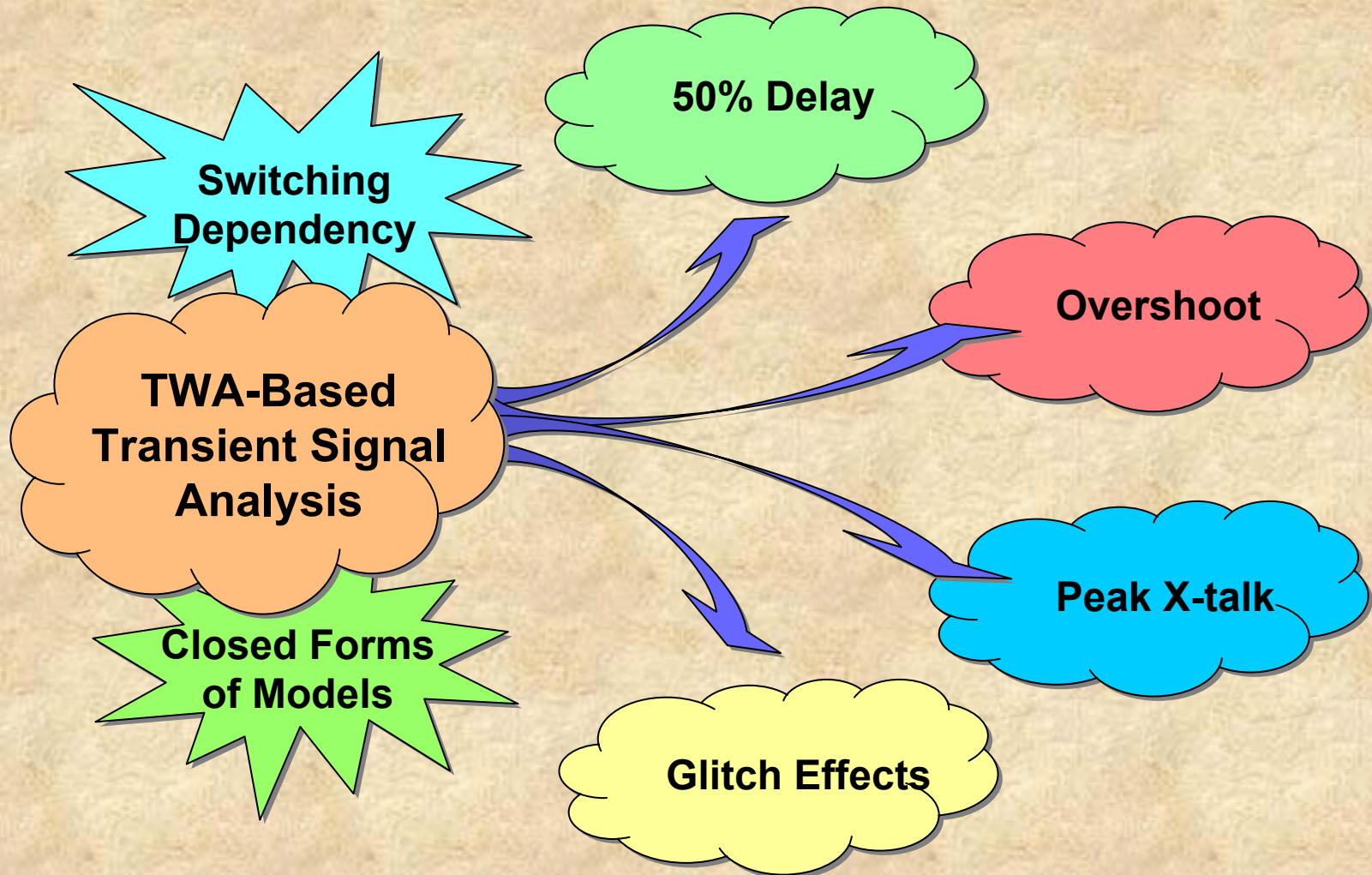
10mm  
0↑0

Crosstalk



10mm  
50Ω / 0.1pF

# TWA-Based Analytical Signal Integrity Models



# Frequency-Domain Response

**Incident Wave**  $[W(x, \omega)] = [S][E][B]$

$[S]$  : Voltage Eigenmatrix

$$[E] = \begin{bmatrix} e^{-\gamma_1 x} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & e^{-\gamma_n x} \end{bmatrix}$$

$$[W(x=0, \omega)] = [v_{s1} \cdots v_{sn}]^T$$

$$[B] = [S]^{-1} [W(x=0, \omega)] = [b_1, b_2, \dots, b_n]^T$$

$$b_k = \sum_{j=1}^n s'_{kj} v_{sj}$$

$$[S]^{-1} = \begin{bmatrix} s'_{11} & s'_{12} & \dots & s'_{1n} \\ s'_{21} & s'_{22} & \dots & s'_{2n} \\ \vdots & \vdots & & \vdots \\ s'_{n1} & s'_{n2} & \dots & s'_{nn} \end{bmatrix}$$

$$W_i(x, \omega) = \sum_{k=1}^n \left( \frac{Z_{0k}}{R_{Si} + Z_{0k}} \right) s_{ik} e^{-\gamma_k x} b_k$$

$$\rightarrow V_i(x, \omega) = \sum_{k=1}^n \left( \frac{Z_{0k}}{R_{Si} + Z_{0k}} \right) s_{ik} b_k \left( e^{-\gamma_k x} + \Gamma_k e^{\gamma_k (x-2\ell)} \right)$$

# TWA-Based Approximation

$$V_i(x, \omega) = \sum_{k=1}^n \left( \frac{Z_{0k}}{R_{Si} + Z_{0k}} \right) s_{ik} b_k \left( e^{-\gamma_k x} + \Gamma_k e^{\gamma_k (x-2\ell)} \right)$$

**TWA for the k-th Mode**

$$V_i(x, \omega) \approx \sum_{k=1}^n s_{ik} b_k Q_k(x, \omega) \leftarrow Q_k(x, \omega) : \text{3-pole approximation function}$$

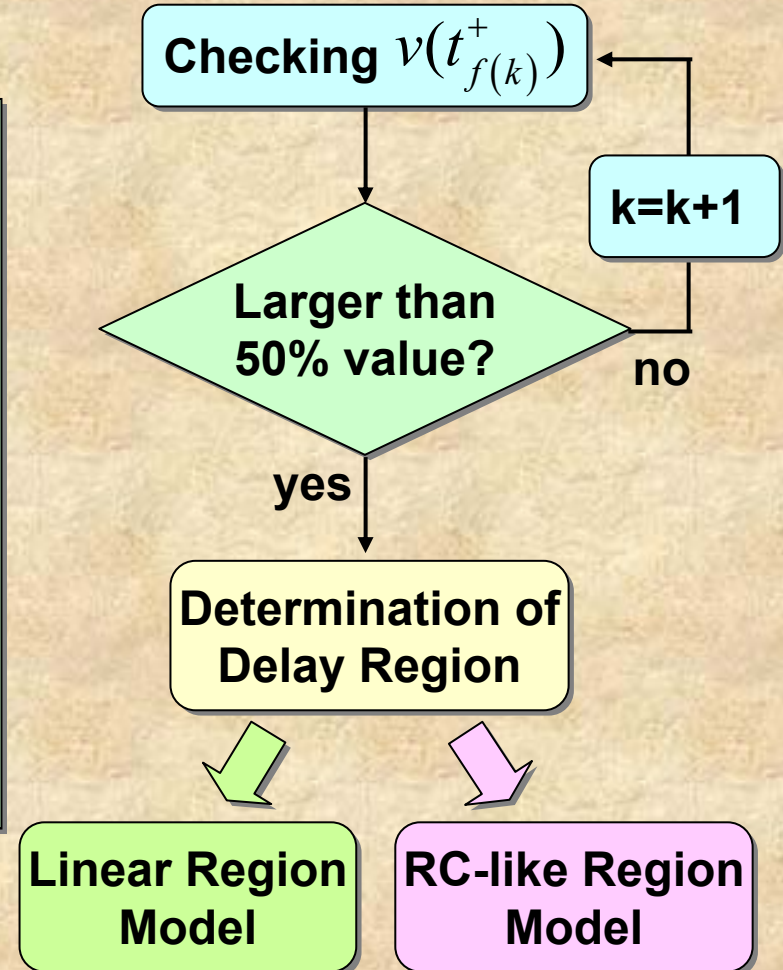
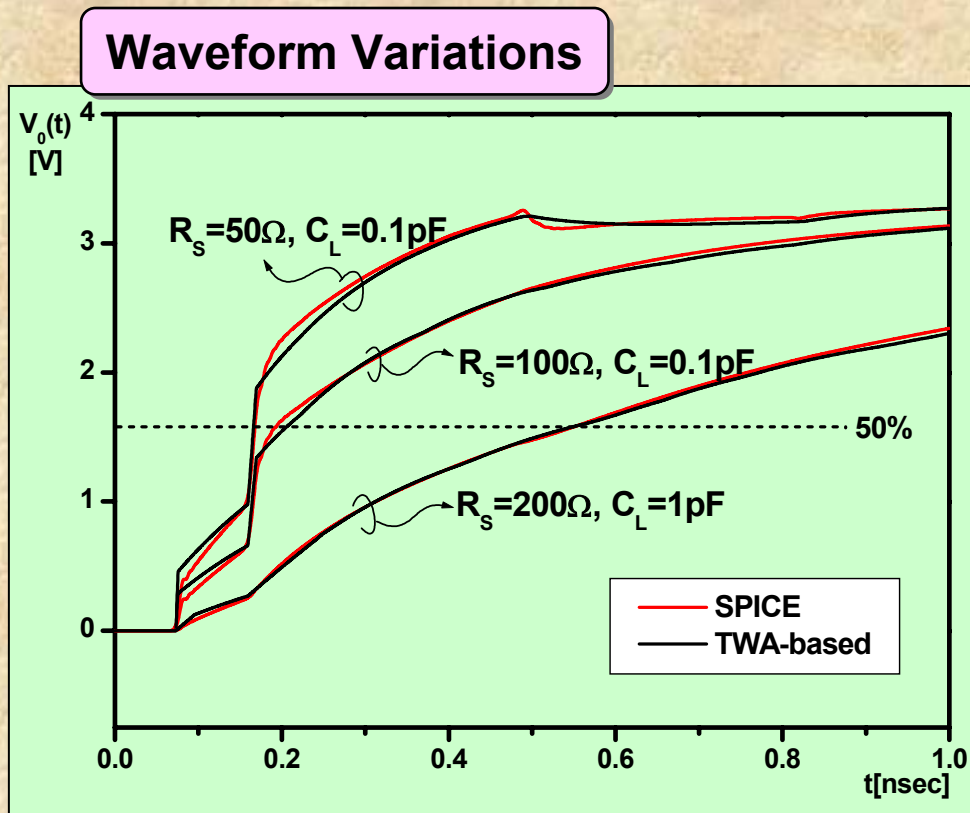
$$v_i(x, t) \approx \sum_{k=1}^n s_{ik} b_k q_k(x, t) \leftarrow q_k(x, t) : \text{time-domain counter part for 3-pole approximation function}$$

$$v_i(x, t) \approx \sum_{k=1}^n s_{ik} b_k p_k(x, t) \leftarrow p_k(x, t) : \text{time-domain approximation function}$$

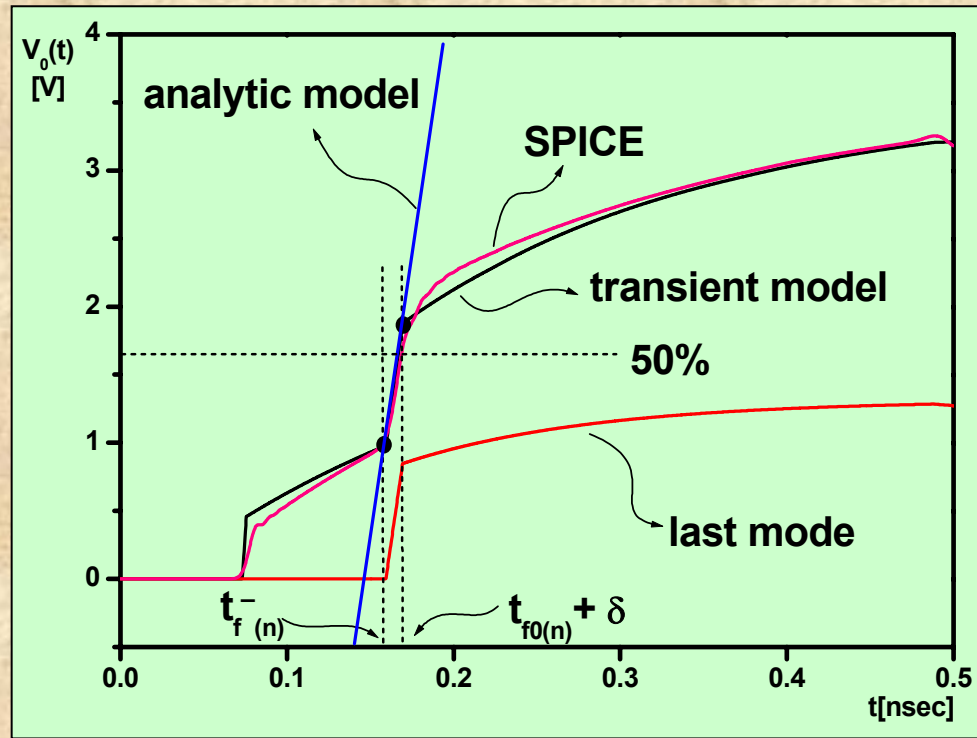
**General Waveform**

$$v_i(x, t) \approx \sum_{k=1}^n \left\{ s_{ik} \left( \sum_{j=1}^n s'_{kj} v_{sj} \right) p_k(x, t) \right\}$$

# Closed Form of Delay Model



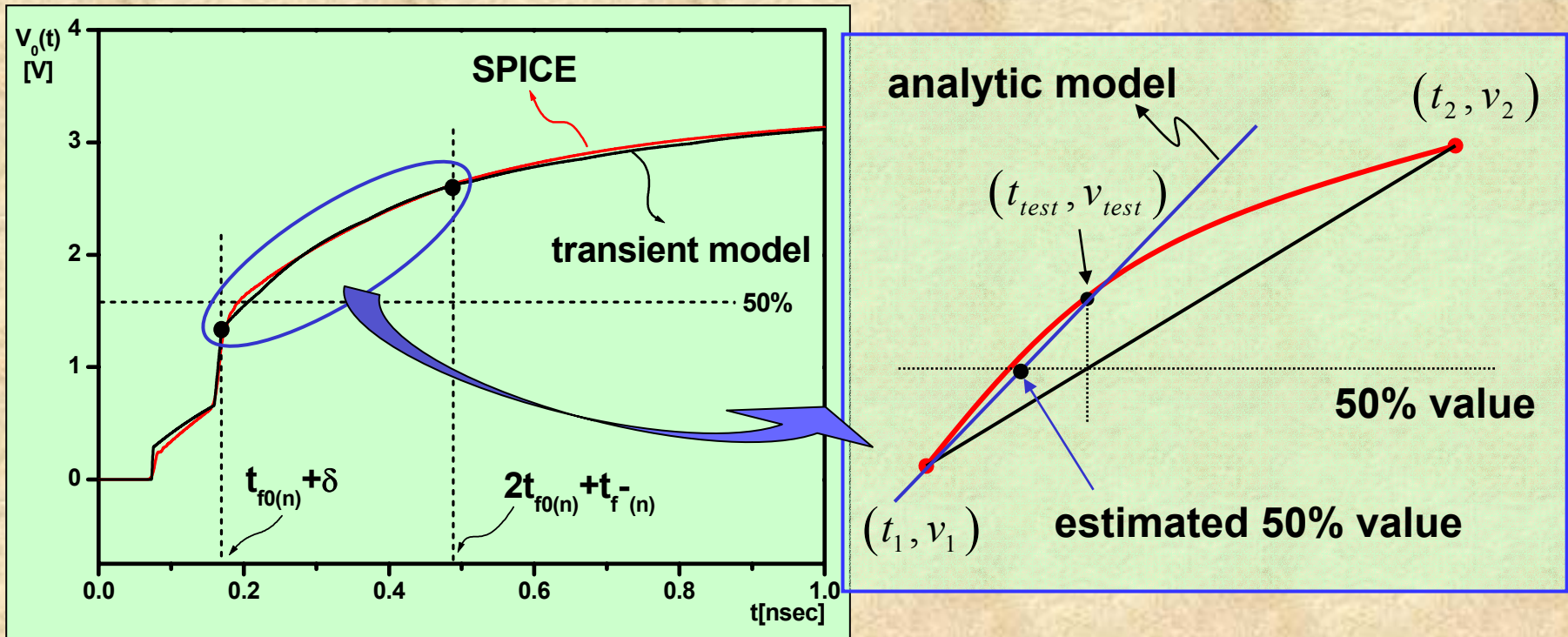
# Delay Model for Linear Region



$$v_i(t) \approx \frac{v_i(t_{f0(n)} + \delta_{(n)}) - v_i(t_{f(n)}^-)}{2\delta_{(n)}} (t - t_{f(n)}^-) + v_i(t_{f(n)}^-)$$

$$t_{50\%delay} = \frac{2\delta_{(n)}}{v_i(t_{f0(n)} + \delta_{(n)}) - v_i(t_{f(n)}^-)} \left( 0.5v_{si} - v_i(t_{f(n)}^-) \right) + t_{f(n)}^-$$

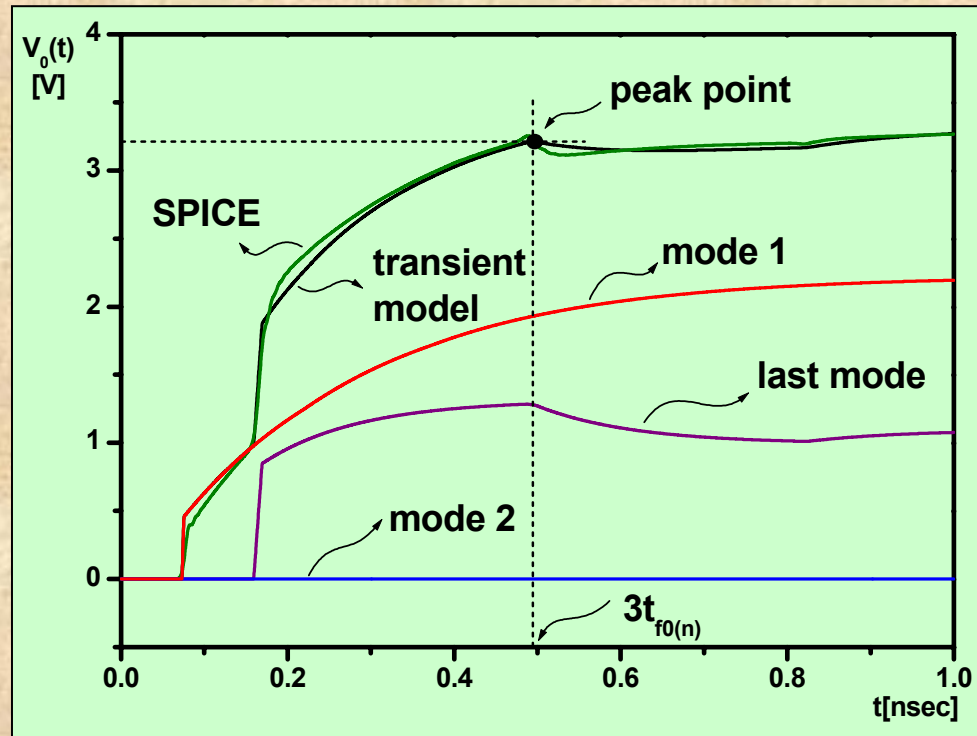
# Delay Model for RC-like Region



$$t'_{50\%delay} = \frac{t_{test} - (t_{f0(n)} + \delta_{(n)})}{v_{test} - v_i(t_{f0(n)} + \delta_{(n)})} \left( 0.5v_{si} - v_i(t_{f0(n)} + \delta_{(n)}) \right) + (t_{f0(n)} + \delta_{(n)})$$

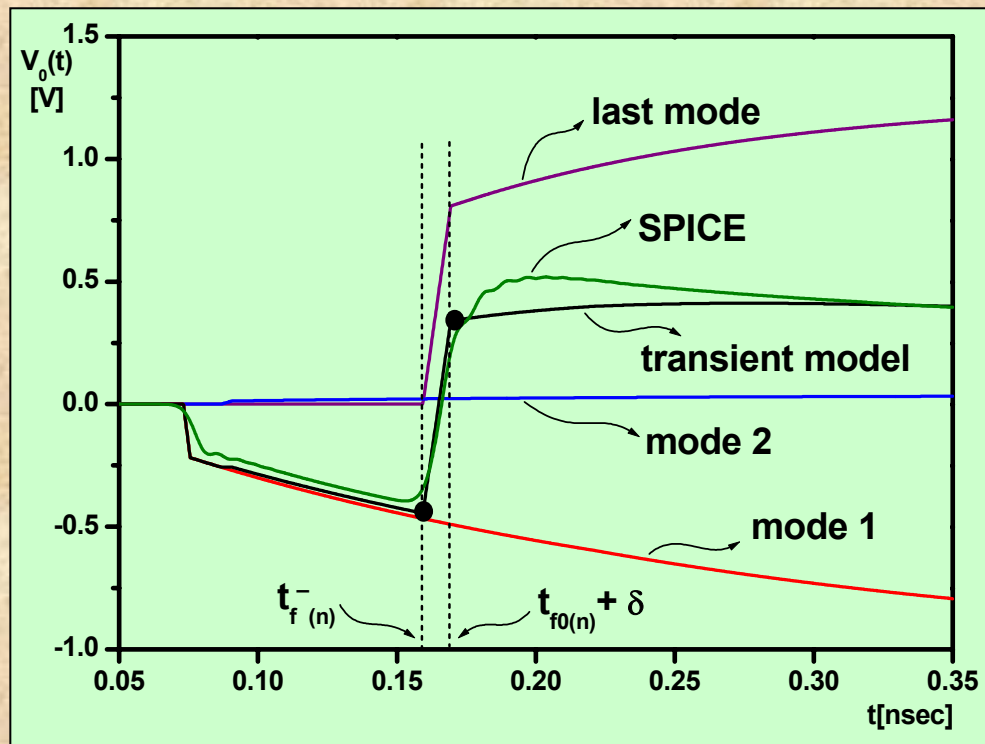
$$v_{test} = v_i(t_{test}) \quad t_{test} = \frac{2t_{f(n)}^-}{v_i(3t_{f0(n)} - \delta_{(n)}) - v_i(t_{f0(n)} + \delta_{(n)})} \left( 0.5v_{si} - v_i(t_{f0(n)} + \delta_{(n)}) \right)$$

# Closed Form of Overshoot



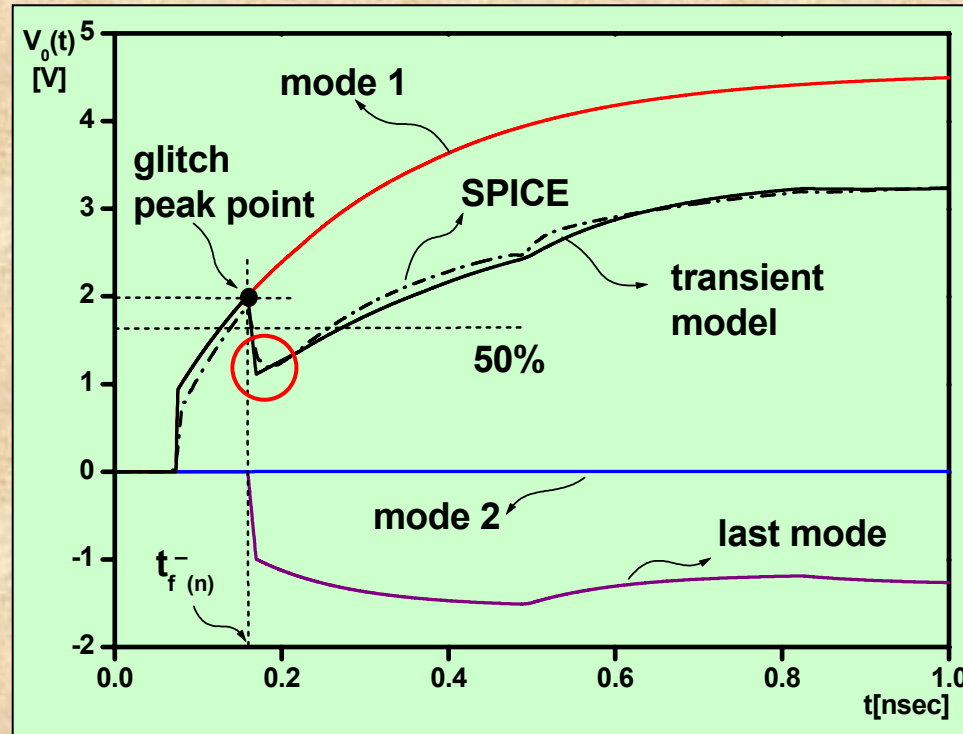
$$v_{i-peak} = \text{MAX} \left( v_{si}, \sum_{k=1}^n \left\{ s_{ik} \left( \sum_{j=1}^n s'_{kj} \cdot v_{sj} \right) \cdot p_k \left( 3t_{f0(n)} \right) \right\} \right)$$

# Closed Form of Peak X-talk



$$\text{MAX} \left( \left| \sum_{k=1}^n \left\{ s_{ik} \left( \sum_{j=1}^n s'_{kj} \cdot v_{sj} \right) \cdot T_k \left( t_{f(n)}^- \right) \right\} \right|, \left| \sum_{k=1}^n \left\{ s_{ik} \left( \sum_{j=1}^n s'_{kj} \cdot v_{sj} \right) \cdot T_k \left( t_{f0(n)}^+ + \delta_{(n)} \right) \right\} \right| \right)$$

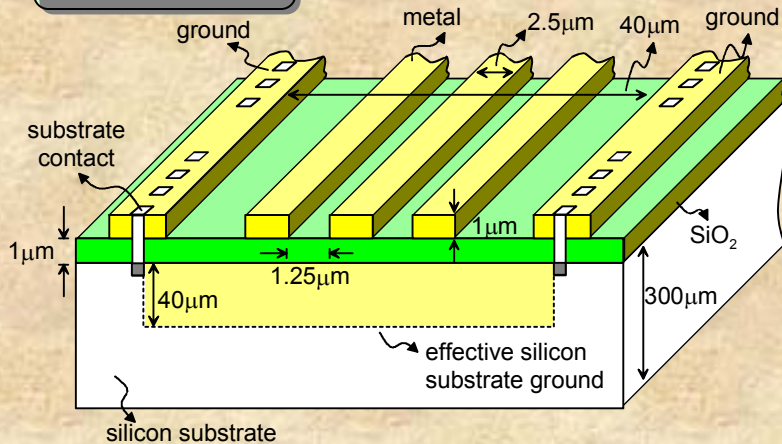
# Closed Form of Glitch Signal



$$v_{i-glitch-peak} = v_i(t_{f(n)}^-) \approx \sum_{k=1}^n \left\{ s_{ik} \left( \sum_{j=1}^n s'_{kj} v_{sj} \right) p_k(t_{f(n)}^-) \right\}$$

# Verification of Analytical Signal Integrity Models

## Structure



## Parameters

$$[L] = \begin{bmatrix} 7.160 & 4.924 & 3.838 \\ 4.924 & 7.010 & 4.924 \\ 3.838 & 4.924 & 7.160 \end{bmatrix} \quad [nH/cm]$$

$$[C] = \begin{bmatrix} 2.225 & -0.522 & -0.042 \\ -0.522 & 2.427 & -0.522 \\ -0.042 & -0.522 & 2.225 \end{bmatrix} \quad [pF/cm]$$

$$[R] = \text{diag}(68.966) \quad [\Omega/cm]$$

## Basic Patterns

Variables Test Item	Line Length	Switching Pattern	Source & Load	Input
Delay	10mm	0↑0	R <sub>S</sub> =50W C <sub>L</sub> =0.1pF	3.3V
X-Talk		↑0↑		
Overshoot		0↑0		
Glitch	10mm	↓↑↓	R <sub>S</sub> =50W, C <sub>L</sub> =0.1pF	3.3V

# Verification Data(1)

## Variable

- Line Lengths

0↑0 Switching

Approximately  
5% Error

Line Length	Active Line Delay [psec]		Active Line Overshoot [V]		Quiet Line Crosstalk [V]	
	SPICE	TWA	SPICE	TWA	SPICE	TWA
1mm	19.2	17.4	3.5913	3.3854	0.5807	0.6821
2mm	35.5	33.7	3.5868	3.4848	0.5762	0.6592
5mm	85.2	83.6	3.4601	3.4139	0.53	0.5652
10mm	168.8	166.8	3.2548	3.2060	0.5192	0.4446

## Variable

- Switching Patterns

Approximately  
2% error

Switching Patterns	Active Line Delay [psec]		Active Line Overshoot [V]		Quiet Line Crosstalk [V]	
	SPICE	TWA	SPICE	TWA	SPICE	TWA
0↑0	168.8	166.8	3.2548	3.2082	0.5192	0.4446
↑↑↑	167.6	165.7	4.0301	3.9860	—	—
↑0↑	170	165.9	3.5262	3.5822	1.0153	1.0275

# Verification Data(2)

## Variables

- Input Driver
- Output Load

0↑0 Switching

Items (driver/load)	SPICE [psec]	TWA-based [psec]	Error[%]
50Ω/0.1pF	168.8	166.8	1.2
100Ω/0.1pF	203.2	219.5	8
200Ω/1pF	582.2	589.3	1.2

## Variables

- Line Lengths

Line Length	↑0↑ crosstalk [V]		↑↑0↑↑ crosstalk [V]	
	SPICE	TWA	SPICE	TWA
1mm	1.176	1.5712	1.6705	2.1156
2mm	1.2022	1.5201	1.7357	2.0934
5mm	1.0829	1.3040	1.6241	1.8696
10mm	1.0153	1.0275	1.3867	1.5456

Approximately  
15% Error  
(Overestimation)

# Verification Data(3)

**Glitch Signal**  
(↓↑↓ Switching)

**Variables**

- Line Lengths

Line Length	Delay [psec]		Glitch Peak [V]	
	SPICE	TWA	SPICE	TWA
1mm	10.7	9.5	2.452	3.0690
2mm	18.9	16.9	2.4743	2.9691
5mm	51	46.4	2.2505	2.5470
10mm	258.4	275.6	1.8435	2.0068

Approximately  
**15% Error**  
(Overestimation)

**Non-Identical Lines**

**Variables**

- Line Width
- Spacing

model items	SPICE	TWA-based	Error[%]
50% delay	158.6 psec	155.7 psec	1.8
Overshoot	3.2265 V	3.2855 V	1.8
Crosstalk	0.5014 V	0.5215 V	4.0

Approximately  
**5% Error**

# Execution Time Computation

**Execution Environment**

SPICE	Model
SUN Ultrasparc-10	AMD Athlon 750MHz

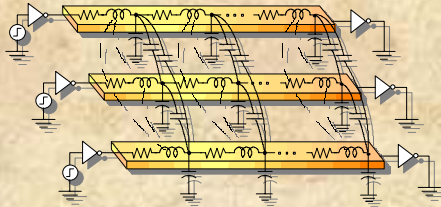
**Execution Time**

Items model	3-lines [sec]	5-lines [sec]	Output
SPICE	78	197	Waveform
Eq. (14)	0.03	0.06	50% delay
Eq. (18)	0.03	0.06	Overshoot
Eq. (19)	0.03	0.06	Crosstalk

 **2500~3000 Times Faster than SPICE !!**

# Summary of the Presentation

## Multiple Lines



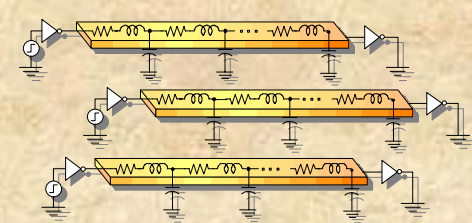
## Frequency Response (Telegrapher Equation)

$$\frac{d^2[V(x)]}{dx^2} = [Z][Y][V(x)]$$

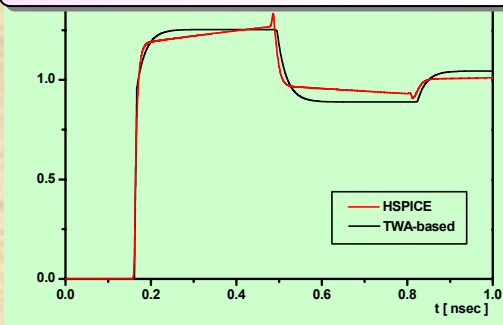
$$[Z] = [R] + s[L]$$

$$[Y] = [G] + s[C]$$

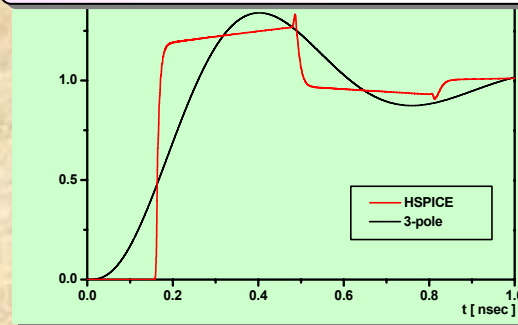
## Modal Decoupling



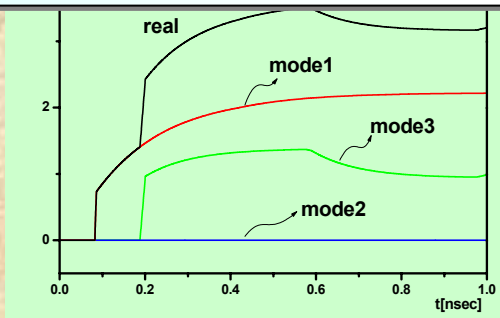
## Time-Domain Approximation



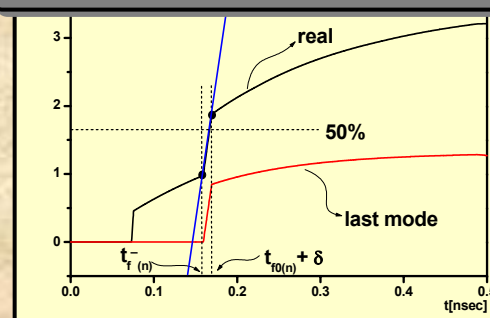
## 3-Pole Approx. In Freq.-Domain



## Analytical Signal Transient Model



## Delay, X-talk, Overshoot, Glitch



## Verification

model items	SPICE	TWA-based	Error [%]
50% delay	158.6 psec	155.7 psec	1.8
Overshoot	3.2265 V	3.2855 V	1.8
Crosstalk	0.5014 V	0.5215 V	4.0

# Conclusion

- ❑ **New Analytical Signal Integrity Models.**
- ❑ **Excellent Agreement with Approximately 5% Error.**
- ❑ **Considered to be a Good Conservative Estimation.**
- ❑ **2500 ~ 3000 times Faster than SPICE Simulation.**