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# **Statistical Crosstalk Aggressor Alignment Aware Interconnect Delay Calculation**

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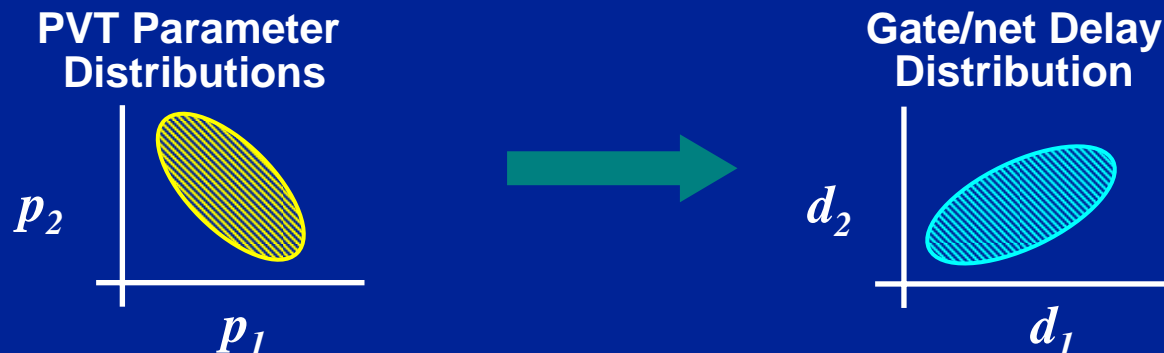
**<http://vlsicad.ucsd.edu>**

# Outline

- **SSTA Background**
- **Problem formulation**
- **Method: theory and implementation**
- **Experiment**
- **Summary**

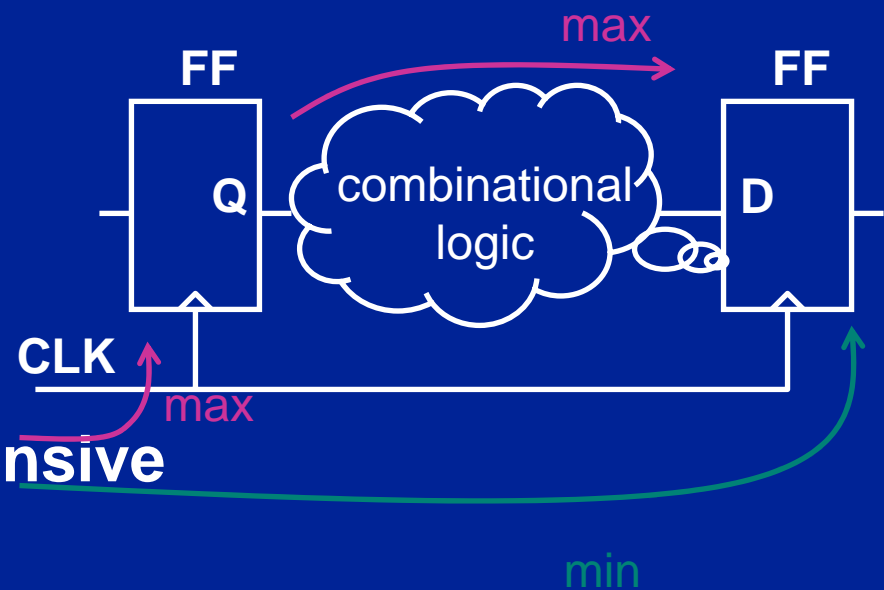
# Background: Variability

- Increased variability in nanometer-scale VLSI designs
  - Process:
    - OPC → Lgate
    - CMP → thickness
    - Doping → Vth
  - Environment:
    - Supply voltage → transistor performance
    - Temperature → carrier mobility  $\mu$  and Vth
- These (PVT) variations result in circuit performance variation



# Background: Timing Analysis

- Min/Max-based
  - ⊙ Inter-die variation
  - ⊙ Pessimistic
- Corner-based
  - ⊙ Intra-die variation
  - ⊙ Computationally expensive
- Statistical
  - ⊙ PDF for delays
  - ⊙ Reports timing yield



# Background: SSTA

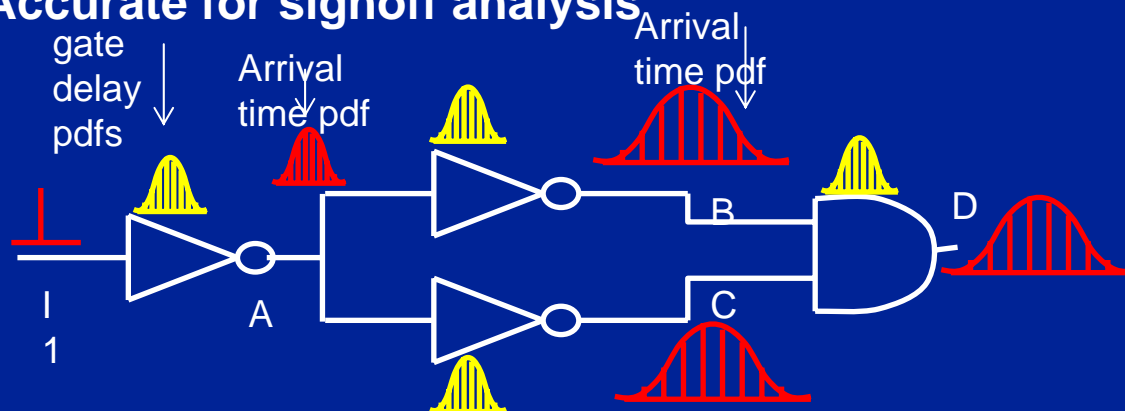
- Represent delays and signal arrival times as random variables

## ⊙ Block-based

- Each timing node has an arrival time distribution
- Static worst case analysis
- Efficient for circuit optimization

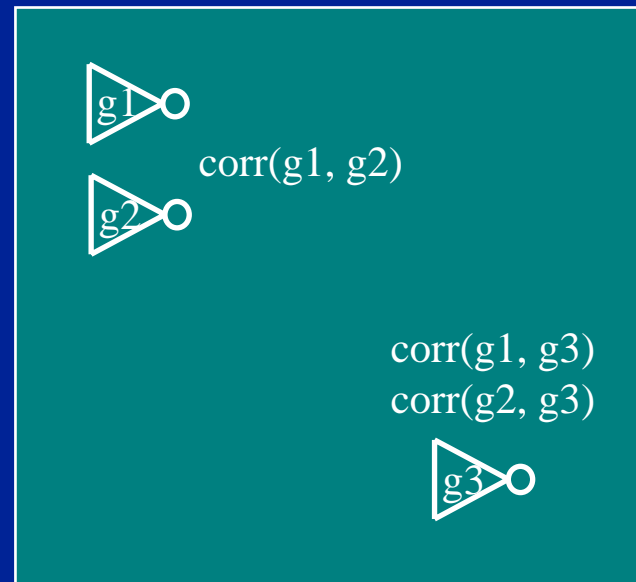
## ⊙ Path-based

- Each timing node *for each path* has an arrival time distribution
- Corner-based or Monte Carlo analysis
- Accurate for signoff analysis



# Background: SSTA Correlations

- Delays and signal arrival times are random variables
- Correlations come from
  - Spatial
    - inter-chip, intra-chip, random variations
  - Re-convergent fanout
    - Multiple-input switching
    - Cross-coupling
    - .....

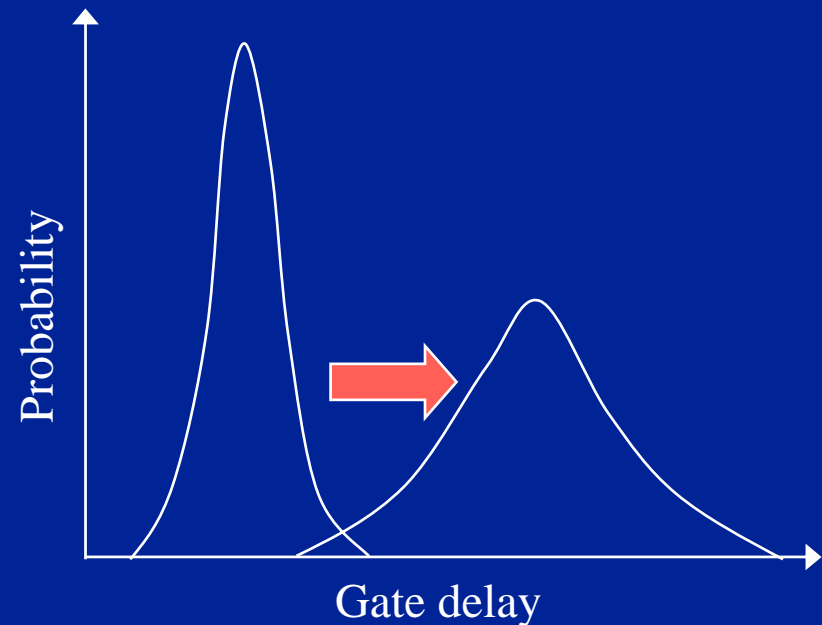
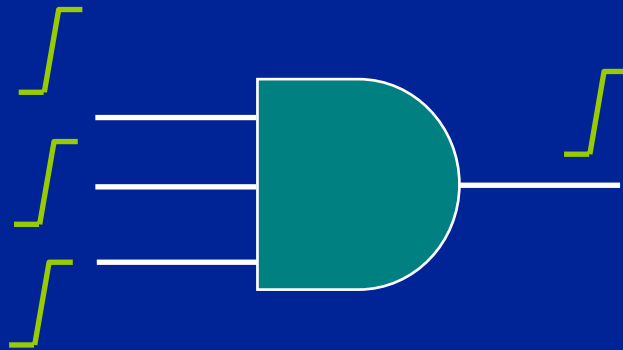


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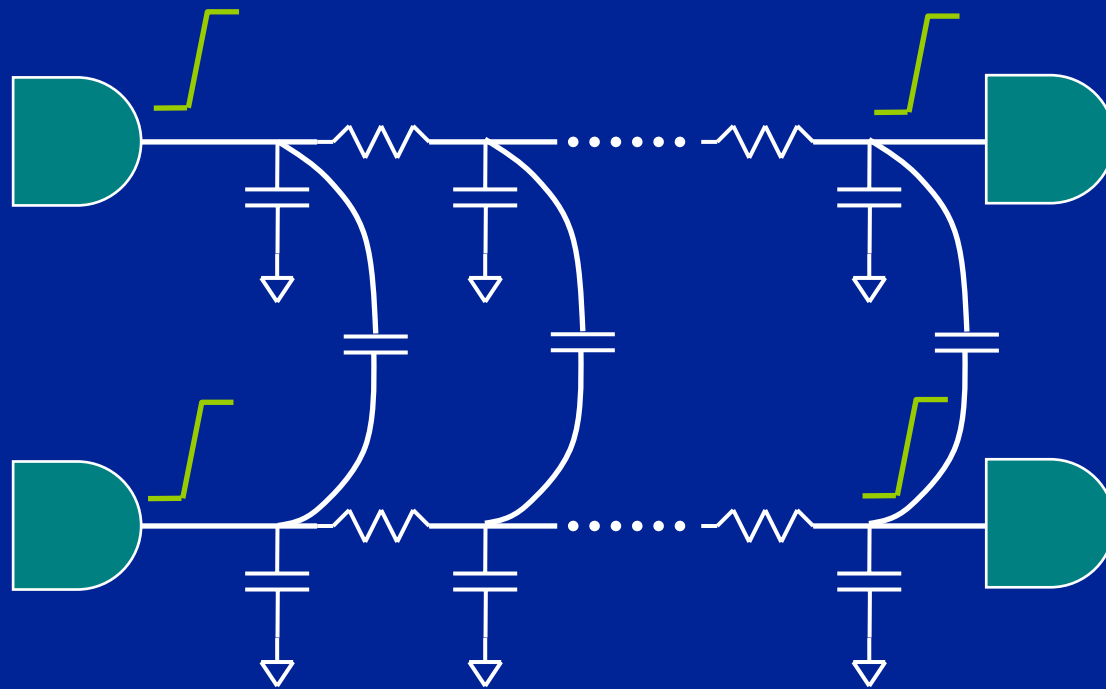
# Multiple-Input Switching

- Simultaneous signal switching at multiple inputs of a gate leads to up to 20%(26%) gate delay mean (standard deviation) mismatch
  - [Agarwal-Dartu-Blaauw, DAC-04]



# Crosstalk Aggressor Alignment

- Crosstalk aggressor alignment induced interconnect delay variation is an equally significant source of uncertainty in SSTA



# Problem Formulation

- **Given**
  - ⊙ Coupled interconnect system
  - ⊙ Input signal arrival time distributions
- **Find**
  - ⊙ Output signal arrival time distributions
- We represent **signal arrival times as polynomial functions of normally distributed random variables**

$$x_i = f_i(r_1, r_2, \dots)$$

$$r_i \sim N(\mu_i, \sigma_i)$$

- E.g., for first order approximation of two input signal arrival times, their skew (crosstalk alignment) is given by normally distributed random variables **with correlation taken into account**

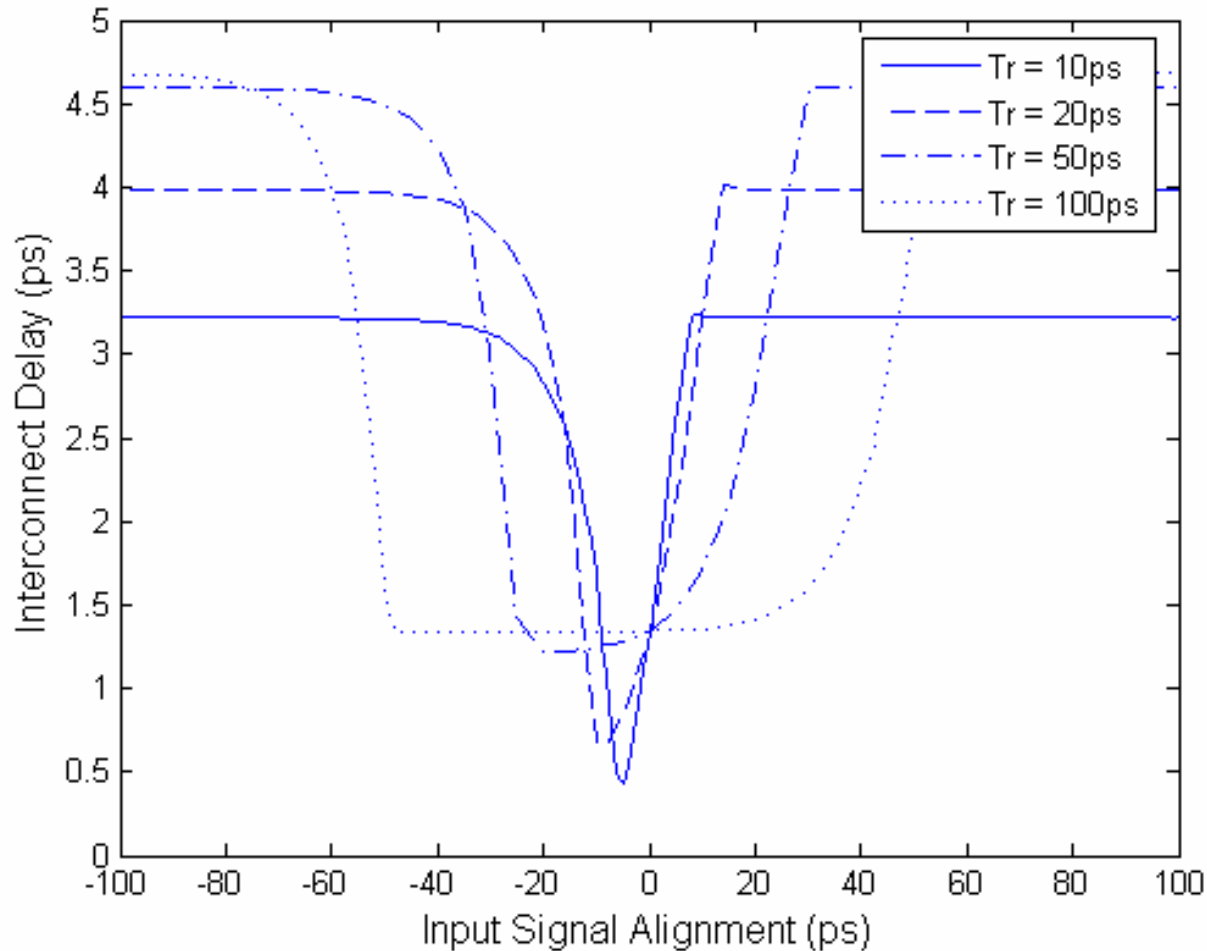
$$x_1 \sim N(\mu_1, \sigma_1) \quad x_2 \sim N(\mu_2, \sigma_2)$$

$$x' = x_2 - x_1 \sim N(\mu' = \mu_2 - \mu_1, \sigma' = (\sigma_1^2 + \sigma_2^2 + corr)^{1/2})$$

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# Interconnect Delay as a Function of Crosstalk Alignment

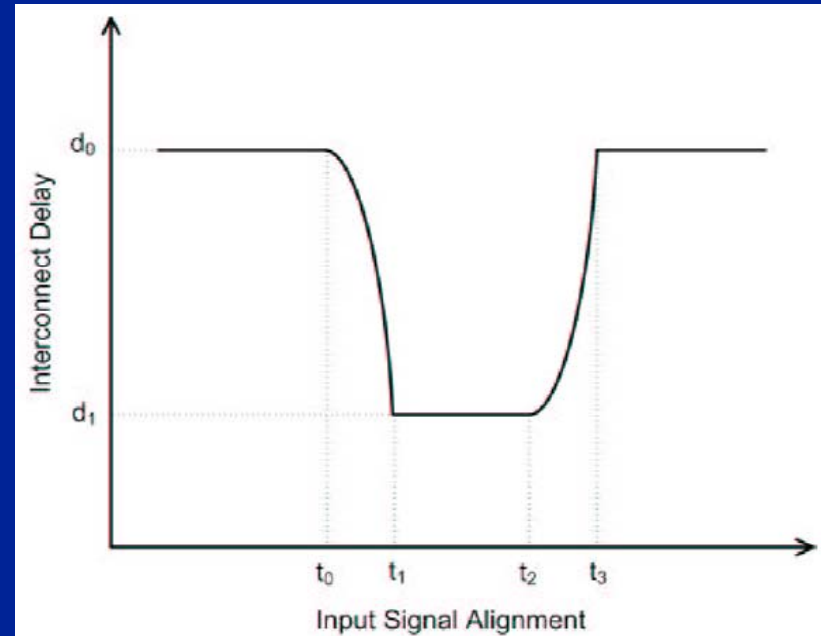


For 1000 $\mu\text{m}$  global interconnects in 90nm technology

# Interconnect Delay as a Function of Crosstalk Alignment

- More complex than the timing window concept
- Can be computed by simulation or delay calculation
- Approximate as a piecewise quadratic function

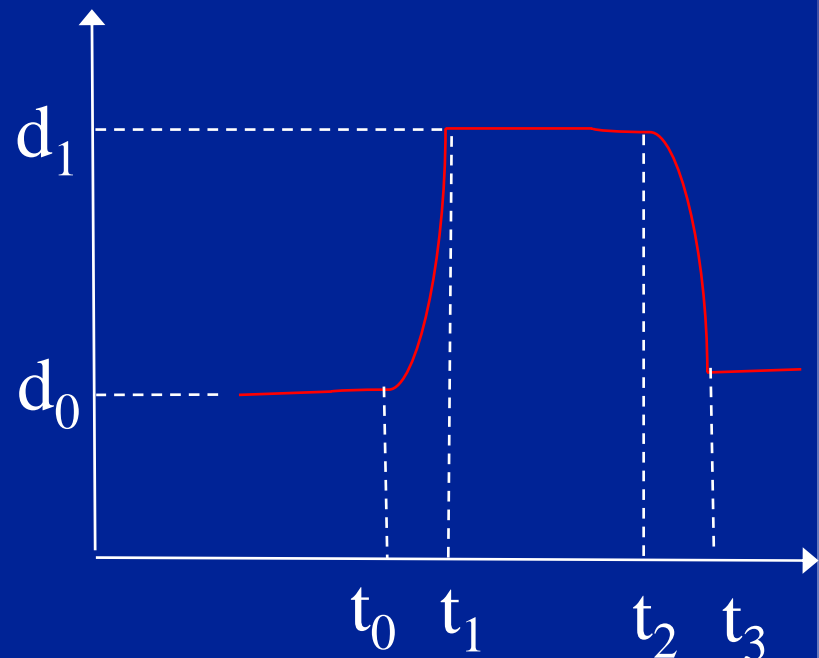
$$\tau = \begin{cases} d_0 & x' \leq t_0 \\ a_0 + a_1 x' + a_2 x'^2 & t_0 \leq x' \leq t_1 \\ d_1 & t_1 \leq x' \leq t_2 \\ b_0 + b_1 x' + b_2 x'^2 & t_2 \leq x' \leq t_3 \\ d_0 & t_3 \leq x' \end{cases}$$



# Interconnect Delay as a Function of Crosstalk Alignment

- Similarly, we can model the case of input signals switching in opposite directions
  - (not discussed here)

$$\tau = \begin{cases} d_0 & x' \leq t_0 \\ a_0 + a_1 x' + a_2 x'^2 & t_0 \leq x' \leq t_1 \\ d_1 & t_1 \leq x' \leq t_2 \\ b_0 + b_1 x' + b_2 x'^2 & t_2 \leq x' \leq t_3 \\ d_0 & t_3 \leq x' \end{cases}$$



# Closed-Form Interconnect Delay Distribution

For a normal distribution of crosstalk alignment

$$P(\tau) = \begin{cases} \frac{1}{2} \left( \operatorname{erf} \left( \frac{t_2 - \mu'}{\sigma'} \right) - \operatorname{erf} \left( \frac{t_1 - \mu'}{\sigma'} \right) \right) & \tau = d_1 \\ -\frac{1}{\sqrt{2\pi}\sigma'} \left( \frac{1}{\sigma_a} e^{-\frac{(-\sigma_a - a_1 - 2a_2\mu')^2}{8a_2^2\sigma'^2}} + \frac{1}{\sigma_b} e^{-\frac{(-\sigma_b - b_1 - 2b_2\mu')^2}{8b_2^2\sigma'^2}} \right) & d_1 \leq \tau \leq d_0 \\ 1 - \frac{1}{2} \left( \operatorname{erf} \left( \frac{t_3 - \mu'}{\sigma'} \right) - \operatorname{erf} \left( \frac{t_0 - \mu'}{\sigma'} \right) \right) & \tau = d_0 \end{cases}$$

$$\sigma_a = \sqrt{4a_2(\tau - a_0) + a_1^2}$$

$$\sigma_b = \sqrt{4b_2(\tau - b_0) + b_1^2}$$

# Closed-Form Interconnect Output Signal Arrival Time Distribution

For a normal distribution of crosstalk alignment

$$\begin{aligned}
 P(y) = & \frac{1}{2\sqrt{2\pi}\sigma_{ya}} e^{-\frac{(y-\mu_{ya})^2}{2\sigma_{ya}^2}} \left( F(y, t_1, a_0, a_1, \sigma_{ya}) - F(y, t_0, a_0, a_1, \sigma_{ya}) \right) + \\
 & \frac{1}{2\sqrt{2\pi}\sigma_{yb}} e^{-\frac{(y-\mu_{yb})^2}{2\sigma_{yb}^2}} \left( F(y, t_1, b_0, b_1, \sigma_{yb}) - F(y, t_0, b_0, b_1, \sigma_{yb}) \right) + \\
 & \frac{P_1(y_1 - d_0)}{2} \left( 2 - \operatorname{Erfc} \left( \frac{t_3 - \mu_2 + y - d_0}{\sqrt{2}\sigma'} \right) + \operatorname{Erfc} \left( \frac{t_0 - \mu_2 + y - d_0}{\sqrt{2}\sigma'} \right) \right) + \\
 & \frac{P_1(y_1 - d_1)}{2} \left( \operatorname{Erfc} \left( \frac{t_3 - \mu_2 + y - d_1}{\sqrt{2}\sigma'} \right) - \operatorname{Erfc} \left( \frac{t_0 - \mu_2 + y - d_1}{\sqrt{2}\sigma'} \right) \right)
 \end{aligned}$$

$$\mu_{ya} = \mu_1 + a_0 - a_1(\mu_1 - \mu_2) \quad \sigma_{ya} = \sqrt{(1 - a_1^2)\sigma_1^2 + a_1^2\sigma'^2}$$

$$\mu_{yb} = \mu_1 + b_0 - b_1(\mu_1 - \mu_2) \quad \sigma_{yb} = \sqrt{(1 - b_1^2)\sigma_1^2 + b_1^2\sigma'^2}$$

# Statistical Delay Calculation for Coupled Interconnects

**Input: Coupled interconnects**

**Input signal arrival time distributions**

**Process variations**

**Output: Output signal arrival time distributions**

- 1. Interconnect delay calculation for sampled crosstalk alignments**
- 2. Approximate interconnect delay in a piece-wise quadratic function of crosstalk alignment**
- 3. Compute output signal arrival time distribution by closed-form formulas**
- 4. Apply superposition for each input**
- 5. Combine with other process variations**

# Multiple Aggressors / Variations

- An RLC interconnect system is a *linear* system, which enables us to apply superposition to model the effects of multiple crosstalk aggressors
- Correlated variation sources
  - For each condition
    - Compute *conditional probabilities*
  - Combine conditional probabilities
- Independent variation sources
  - *Superposition*

# Runtime Analysis

- *Interconnect delay calculation for  $N$  sampled crosstalk alignment takes  $O(N)$  time*  
where  $N = \min(1.5 * \text{max input transition time}, 6 * \text{sigma of crosstalk alignment}) / \text{time\_step}$
- Fitting takes  $O(N)$  time
- Computing output signal arrival time distribution takes constant time, e.g., when updating statistical delay calculation in an iterative SSTA

# Iterative SSTA

- STA-SI goes through an *iteration of timing window refinement for reduced pessimism* of worst case analysis
- SSTA-SI goes through an *iteration of signal arrival time pdf refinement with reduced deviations*

# Outline

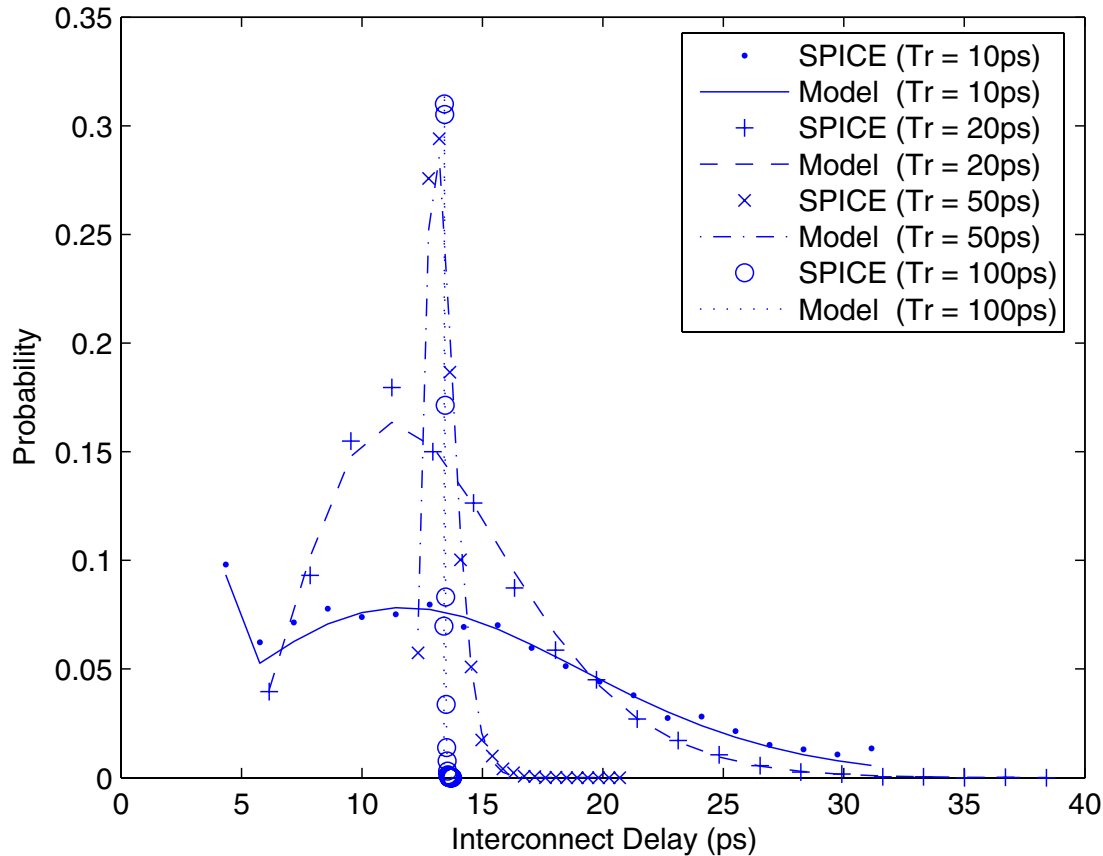
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# Experimental Setting

- Quadratic function regression performed with Origin
- Monte Carlo simulation performed with HSPICE
- Closed-form model-based distribution calculation: C
- Testcases:
  - ⊙ 70nm BTM model (Cao et al.)
  - ⊙ 130nm industry designs

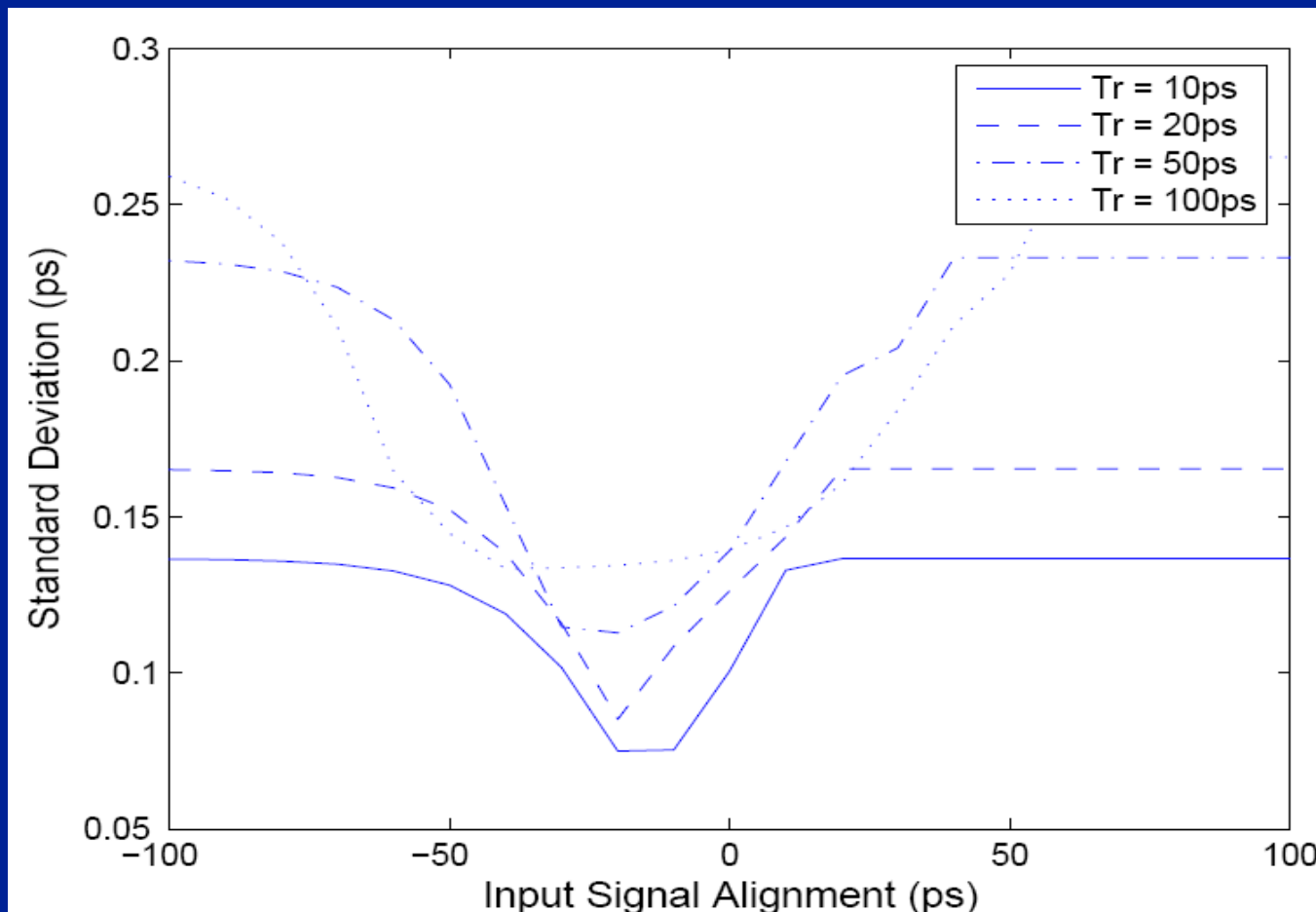
70nm	L (um)	W(um)	S(um)	T(um)
global	1000	0.45	0.45	1.20
intermediate	200	0.14	0.14	0.35
local	30	0.10	0.10	0.20

# Interconnect Delay Distribution



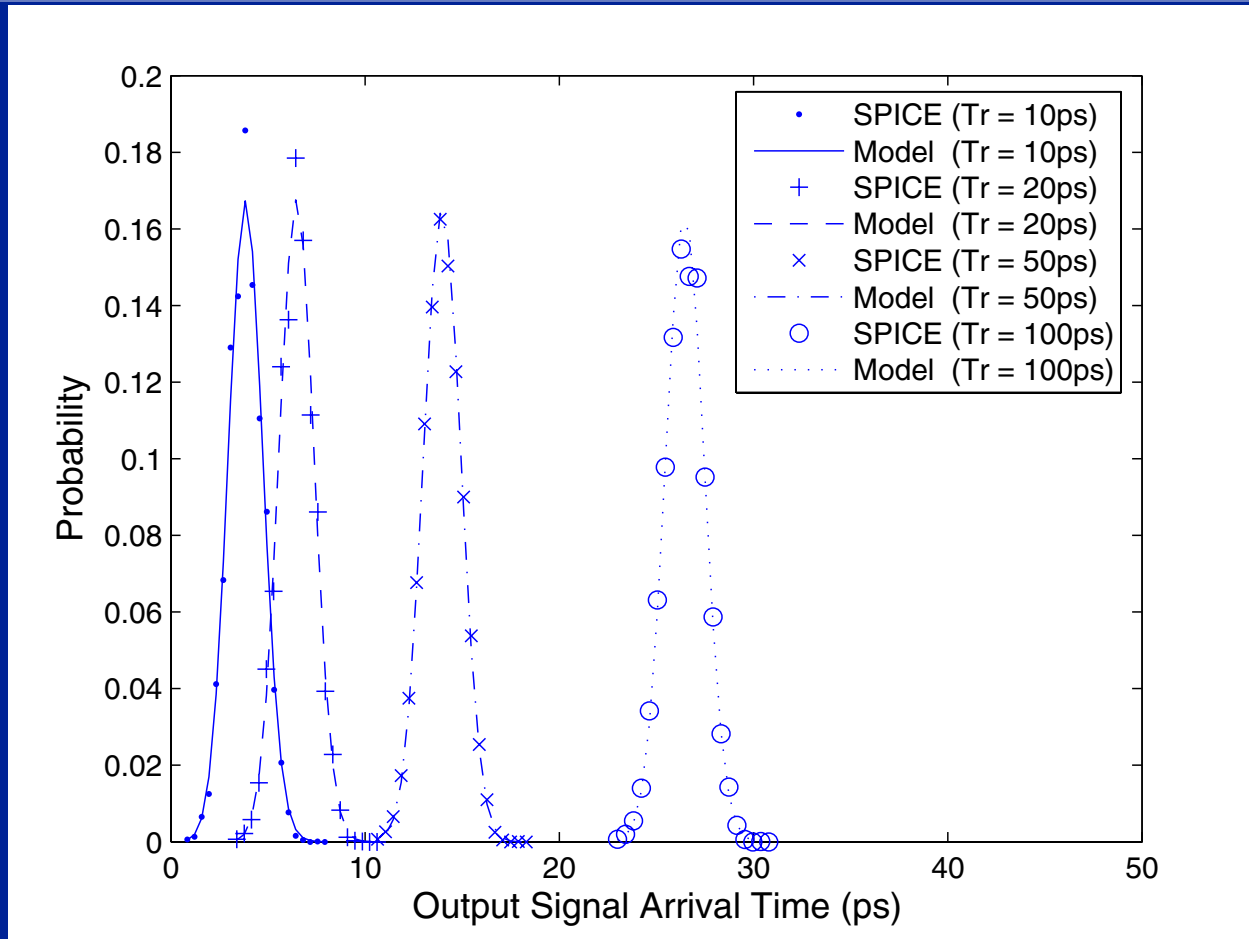
For a pair of 1000um coupled global interconnects in 70nm BPTM technology, with 10, 20, 50 and 100ps input signal transition time, and crosstalk alignment in a normal distribution  $N(0, 10ps)$

# Interconnect Delay Standard Deviation due to Varied Wire Width



For a pair of 1000um coupled global interconnects in 70nm BPTM technology, with 10, 20, 50 and 100ps input signal transition time, and wire width variation in a normal distribution  $N(0, 10\%)$

# Interconnect Output Signal Arrival Time Distribution



For a pair of  $1000\mu\text{m}$  coupled global interconnects in 70nm BPTM technology, with 10, 20, 50 and 100ps input signal transition time, and crosstalk alignment in a normal distribution  $N(0, 6\text{ps})$

# Experiment

Test case 1: 1000 $\mu$ m interconnects of 70nm BPTM technology

	Delay				Output					
	SPICE		Model		SPICE		Model		% diff	
3 $\sigma$ Tr(ps)	$\mu$	$\sigma$	$\mu$	$\sigma$	$\mu$	$\sigma$	$\mu$	$\sigma$	$\mu$	$\sigma$
50 50	3.83	0.85	3.82	0.83	29.4	16.2	29.7	16.6	0.78	2.46
100 100	3.82	0.92	3.84	0.83	54.8	32.8	55.6	33.9	1.52	3.38
200 200	3.78	0.96	3.78	0.82	105.2	65.9	106.3	67.0	1.06	1.65

Test case 2: interconnects in a 130 $\mu$ m industry design

	Delay				Output					
	SPICE		Model		SPICE		Model		% diff	
3 $\sigma$ Tr(ps)	$\mu$	$\sigma$	$\mu$	$\sigma$	$\mu$	$\sigma$	$\mu$	$\sigma$	$\mu$	$\sigma$
50 100	4.29	0.16	4.30	0.15	54.5	16.4	53.4	16.1	-2.09	-0.05
100 100	4.30	0.18	4.30	0.17	54.8	32.9	54.1	33.0	-0.17	0.18
200 200	4.25	0.18	4.25	0.16	105.2	65.9	104.9	66.1	-0.28	0.35

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# Summary

- ***SSTA must consider SI effects!***
- We take crosstalk aggressor alignment into account in statistical interconnect delay calculation
  - We approximate interconnect delay as ***a piecewise quadratic function*** of crosstalk aggressor alignment
  - We derive ***closed-form formulas*** for interconnect delay and output signal arrival time distribution for given input signal arrival times in polynomial functions of normal distributions
  - Our experiments show that neglecting crosstalk alignment effect could lead to up to ***114.65% (71.26%)*** mismatch of interconnect delay means (standard deviations), while our method gives output signal arrival time means (standard deviations) within ***2.09% (3.38%)*** of SPICE results

**Thank you !**

# Experiment: Delay Variation

	SPICE		Without Crosstalk		Our	
Tr =10ps	$\mu$	$\sigma$	$\mu$ Diff(%)	$\sigma$ Diff(%)	$\mu$ Diff(%)	$\sigma$ Diff(%)
$\mu' = -10ps$	3.13	0.38	5.65 80.5	0.17 -56	3.12 -0.64	0.38 -0.1
$\mu' = 0ps$	3.38	0.58	5.63 66.5	0.17 -71	3.38 0.0	0.57 -1.9
$\mu' = 10ps$	4.85	0.31	5.64 16.3	0.17 -47	4.85 0.0	4.32 -0.23
Tr =20ps	$\mu$	$\sigma$	$\mu$ Diff(%)	$\sigma$ Diff(%)	$\mu$ Diff(%)	$\sigma$ Diff(%)
$\mu' = -10ps$	3.14	0.37	6.52 114	0.19 -47	3.12 -0.96	0.37 1.26
$\mu' = 0ps$	3.67	0.47	6.53 77.9	0.19 -58	3.66 -0.01	0.47 -1.0
$\mu' = 10ps$	5.22	0.55	6.52 24.9	0.19 -65	5.22 0.0	0.54 -1.48

Test case: 1000 $\mu$ m interconnects of 70nm BPTM technology

Assume: wire width distribution  $N(1, 0.05)$  \* Normal\_width

crosstalk alignment distribution  $N(\mu', 3.33ps)$