What is Rent's Rule?

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When I first heard of the straight line log-log plot of Rent of termincal count versus circuit size, with an exponent of 2/3, I felt that we had a quantitative handle in the field of design automation. One could make an estimate of average wirelength, which actually impressed people, who had to look into the future. We did a lot of wirability analysis with that model. The late Bill Heller, my very good friend, urged me to get a statistical distribution from this, which I did do.

I wondered how one could model Rent's Rule - and to my great disappointment I found that random graphs did not obey Rents Rule. the ratio of the number of connections cut by a bisection to the total number of connections can be proved to approach a non-zero value as the size of the graph increases, while the same ratio will go to a limit of zero for graphs obeying Rent's Rule, independent of exponent. I developed a model for a hierarchical design process, where, if you put in self-similarity, you can derive Rent's Rule - and the Rent exponent can be between 0 and 1. One can determine that the larger the exponent, the smaller is the latency of the graph (i.e. if the design process develops a combinatorial graph, the number of logic books on a path from inputs to outputs would increase as the number of gates taken to the power 1-p, where p is the Rent exponent). One can also find that the log of the number of such graphs is roughly proportional to the number of connections - and that the proportionality constant is roughly independent of the exponent.

Of course we know that there exist designs which behave very much unlike Rents Rule. The worst offenders are, e.g., the crossbar switch, where the number of cut connections is proportional to the number of signals being switched and memory, where the number of cut connections is the log of the number of elements.

The net of this is that Rent Rule is an effect of the design process, and what we see is an average behavior. It should be possible to control the process such that we can, in advance, determine what the Rent exponent should be and what the consequence is of having a given Rent exponent. We have a long way to go.